# Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach*

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**Abstract**

We present a tractable New Keynesian model with incomplete asset markets and labor market frictions. A key feature of the model is endogenous idiosyncratic earnings risk, which may be procyclical or countercyclical. In the latter case –which we argue is empirically more plausible– high risk of unemployment stimulates precautionary saving amongst the employed during recessions. This feature induces a powerful supply-demand feedback mechanism with profound implications. First, a new high-unemployment steady state can arise. Second, monetary policy needs to be more aggressive than dictated by the Taylor principle to avoid local indeterminacy. Third, shocks are amplified and productivity increases may be inflationary. Fourth, the Zero Lower Bound may be inflationary and paradoxical outcomes may be overcome.

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# 1 Introduction

The New Keynesian (NK) model has gained widespread use both in academic research and in policy circles. The crux of the model is that nominal frictions induce inefficient fluctuations in the economy, which monetary and fiscal policies can be designed to address. Clarida, Galí and Gertler (1999) provide a summary of the model's key insights, based on a highly intuitive, three-equation version. Paradoxically, however, these insights do not pertain to unemployment and distributional issues, two central aspects of many policy discussions which are widely considered to be important determinants of aggregate demand.

Recently, a new generation of NK models that addresses these deficiencies has emerged. For example, Gertler and Trigari (2009), Blanchard and Galí (2010), Ravenna and Walsh (2011), and Christiano, Eichenbaum and Trabandt (2016) introduce unemployment by incorporating Search and Matching (SAM) frictions in the labor market. Others have introduced financial market incompleteness, generating inequality in income, wealth and consumption. Kaplan, Moll and Violante (2018) have dubbed such models Heterogeneous Agents New Keynesian (HANK) models. By giving centre stage to HANK and SAM, the new models mark a clear break with the traditional "representative agent" assumption, offer a rich array of cross-sectional predictions, and allow inequality across households to matter in models of aggregate fluctuations, see e.g. McKay and Reis (2016a), Guerrieri and Lorenzoni (2017), and many others.[1]

This paper complements the new vintage of NK models with an analytically tractable counterpart that is as simple as the model in Clarida, Galí and Gertler (1999), but nonetheless features search and matching frictions in the Diamond-Mortensen-Pissarides tradition, and incomplete markets à la Bewley, Huggett and Aiyagari. Our main purpose is to revisit core qualitative results highlighted in the New Keynesian literature and to understand how these results are affected by the interactions between HANK and SAM. A key feature of our framework is that the severity of idiosyncratic earnings risk is endogenously determined, in conjunction with aggregate demand. We demonstrate profound implications of this endogeneity for short- and long-run equilibrium determination, the response of the economy to shocks, and the implications of the Zero Lower Bound (ZLB) on the nominal interest rate.

In our model, households face unemployment risk and search for jobs in a matching market.

[1] Other studies in this new vein include Auclert (2016), Bayer, Pham-Dao, Luetticke and Tjaden (2015), Beaudry, Galizia and Portier (2017), Berger, Dew-Becker, Schmidt and Takahasi (2016), Braun and Nakajima (2012), Challe, Matheron, Ragot and Rubio-Ramirez (2017), den Haan, Rendahl and Riegler (2016), Heathcote and Perri (2015), Gornemann, Kuester and Nakajima (2016), Kekre (2016), Luetticke (2015), McKay, Nakamura and Steinsson (2016), McKay and Reis (2016b), and Ravn and Sterk (2017).

They lack unemployment insurance but can self-insure through savings. Firms produce output using input of labor. New hires are obtained by posting vacancies. Firms are monopolistically competitive but face nominal rigidities when setting prices. There is also a central bank which sets the nominal interest through an interest rate rule.

The model's tractability derives from a limit on participation in the equity market, combined with a borrowing limit in the bond market. These assumptions give rise to an equilibrium with three distinct groups of households: borrowing-constrained unemployed households, unconstrained but asset-poor employed households, and asset-rich but liquidity-constrained households.[2] We can characterize the equilibrium outcomes analytically, which facilitates a clear understanding of the underlying economic mechanisms. Moreover, our analytical approach allows us also to address potential equilibrium multiplicity which may easily be overlooked when solving incomplete-markets models numerically. This possibility is not a mere technical artefact as fluctuations driven by "animal spirits" can arise naturally under incomplete markets and endogenous employment risk.

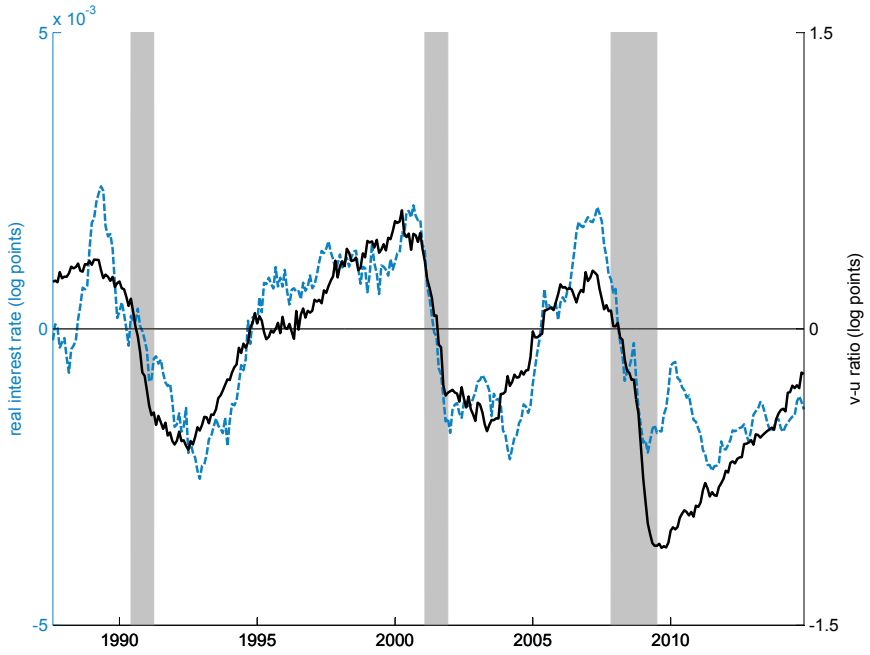
A central feature of the model is the emergence an 'endogenous risk wedge' in the consumption Euler equation.[3] Werning (2015) similarly highlights this type of wedge in an analytical 'aggregated' Euler equation, but does not model explicitly how it is determined in equilibrium. We demonstrate how, in the presence of search and matching frictions, the wedge is pinned down by the tightness of the labor market, which moves over the business cycle. When the economy enters a recession, two opposing forces arise. On the one hand, worsening labor market conditions make new jobs harder to find for job losers. This mechanism introduces *countercyclical earnings risk* because of lack of unemployment insurance. On the other hand, wages tend to fall in recessions, which makes job loss less costly and hence induces *procyclical earnings risk*. We characterize the conditions under which either of these two sources of cyclicality dominates, and relate them to primitives such as wage flexibility, the income loss in terms of unemployment, labor market flexibility, etc.

Which of these sources is likely to dominate in practice? In models with complete markets, agents have an intertemporal savings motive which induces a negative comovement between real

[2]Our setup extends earlier work deriving tractability from assumptions on the borrowing limit, see Krusell, Mukoyama and Smith (2011), Werning (2015), McKay, Nakamura and Steinsson (2017), McKay and Reis (2016b), Bilbiie (2017) and Ravn and Sterk (2017). In these models, agents are unable to borrow. In our analysis, the employed households – who end up pricing the bonds– are in principle able to borrow but choose voluntarily not to do so.

[3]Like Clarida, Galí and Gertler (1999), we abstract from physical capital for reasons of tractability in the main text but look at a version of the model with capital in Appendix A.7.

Figure 1: Real interest rate ($R^r$) and labor market tightness ($v/u$) in the data.




Notes: Real interest rate and labor market tightness (vacancy-unemployment ratio) in the United States; deviations from trend. The real interest rate is expressed on a monthly basis and is computed as the Federal Funds rate minus a six-month moiving average of CPI inflation. Vacancies are measured as the composite Help Wanted index from Barnichon (2010). Data series were logged and de-trended using a linear trend estimated over the period up to the end of 2007.

interest rates and labor market slackness because agents wish to save when jobs are easy to find and income is high. This tendency for negative comovement between real interest rates and job finding prospects is even stronger under incomplete markets when the procyclical earnings risk channel dominates. On the other hand, if earnings risk is countercyclical (when unemployment risk dominates), agents have a strong precautionary savings motive in recessions, which induces a potentially positive comovement between job finding rates and real interest rates. Figure 1 illustrates the relationship between real interest rates and labor market tightness (the ratio of job vacancies to unemployment) in the U.S. These two variables comove positively, indicating that real interest rates are low when jobs are hard to find and vice versa. This points towards dominance of the countercyclical endogenous earnings risk channels. We further carry out back-of-the-envelope calculations which support this conclusion unless wages are extremely procyclical and income losses in case of unemployment are minor.

Next, we explore the implications of endogenous earnings risk, in the presence of incomplete

markets and endogenous rigidities. Countercyclical earnings risk introduces an amplification mechanism, due to a demand-supply side interaction. Intuitively, a worsening of labor market conditions increases unemployment risk, which motivates households to build more precautionary savings. This reduces aggregate goods demand and increases the demand for bonds, pushing down the real interest rate. Because of nominal rigidities, firms respond to lower goods demand by cutting back on new hires. As a result, labor market conditions worsen further creating even more earnings risk, even lower goods demand, and so on. Conversely, if endogenous earnings risk is procyclical, aggregate fluctuations are stabilized. When earnings risk is acyclical, either because the two mechanisms exactly cancel out or because earnings risk is exogenous, the model's implications for aggregate fluctuations are similar to those of standard (two-agent) New Keynesian model (see Debortoli and Galí, 2017).

We highlight a number of key implications of the endogenous risk channel for the macro economy and for monetary policy.[4] The first concerns the steady-state properties of the model. As in the basic NK model, there is an "intended" steady-state equilibrium as well as an unintended "liquidity trap". In the latter steady state, the ZLB binds and output is relatively low, as in Benhabib, Schmitt-Grohé and Uribe (2001, 2002). Unlike the standard NK model, however, our model may have a third steady state, which we label the "unemployment trap." This equilibrium can arise when the endogenous earnings risk is countercyclical and sufficiently strong. In the unemployment trap, aggregate demand is depressed to a level at which it is no longer profitable for firms to invest in vacancies, and in which inflation is moderately lower than in the intended steady state. Therefore, hiring declines to a minimum, which perpetuates high unemployment risk and hence low demand.

We then study local determinacy properties and derive an analytical local determinacy condition for the intended steady state. Countercyclical earnings risk implies that local indeterminacy can arise even when the "Taylor Principle" is satisfied (see e.g. Woodford, 2003, Chapter 2). Intuitively, monetary policy must not only rule out local indeterminacy due to nominal rigidities, but also address the demand-supply interaction. Additionally, the unemployment trap is shown to be determinate under a standard rule which responds more than one-for-one to inflation. Around this steady state, the monetary policy rule determines the rate of inflation, but has no grip on unemployment.

Third, we study the responses of the economy to productivity shocks and monetary policy shocks in the vicinity of the intended steady state. We solve analytically for the dynamic

[4] McKay and Reis (2016b) study optimal social insurance policy in a similar environment.

equilibrium and characterize the amplification (dampening) mechanism due to countercyclical (procyclical) earnings risk. We also show that, under countercyclical risk, positive productivity shocks may increase rather than decrease inflation, due to the ensuing boom in aggregate demand. Via the same mechanism, endogenous risk impacts on the effects of monetary policy shocks. Countercyclical risk makes monetary policy shocks more powerful. Similarly, the systematic component of monetary policy becomes more important as it can counteract the amplification mechanism, by cutting interest rates when demand is low and earnings risk is high. Importantly, the endogenous earnings risk channel that underlies these results is not present in the representative-agent New Keynesian models, nor in two-agent versions of that model, as in for example Galí, López-Salido and Vallés (2003), Bilbiie (2008), and Broer et al. (2016). In other words, market incompleteness matters for the business cycle, provided that earnings risk is endogenous.

Fourth, we revisit the role of the ZLB. In particular, we present a condition under which a *negative* productivity shock moves the nominal interest rate towards the ZLB and show that ZLB episodes are not necessarily deflationary. This happens as the real interest rate declines when unemployment increases, due to a heightened demand for precautionary savings. At the ZLB, a decline in the real interest rate implies an increase in inflation via the Fisher relation. Additionally, we revisit paradoxical properties of the representative-agent NK model which occur when the ZLB binds, see e.g. Eggertsson and Krugman (2012) and Werning (2012). Specifically, we show that our model can overturn the paradox that, at the ZLB, positive productivity shocks may be contractionary, because higher productivity dampens the precautionary savings motive. Our analysis complements McKay, Nakamura and Steinsson (2016), who study the implications of incomplete markets for the "forward guidance puzzle" at the ZLB.

The remainder of this paper is organized as follows. Section 2 presents the model. In Sections 3 and 4 we study, respectively, steady states and local fluctuations. Section 5 focuses on the role of the ZLB. Finally, in section 5 we discuss our model in light of the data, and argue that countercyclical sources of earnings risk likely dominate procyclical forces.

# 2 The Model

We construct a model which combines nominal rigidities in price setting, as in the NK tradition, with labor market matching frictions in the Diamond-Mortensen-Pissarides tradition, and incomplete asset markets in the Aiyagari-Bewley-Huggett tradition. The economy is made up

of households who consume and work, firms which produce output, and a monetary authority in charge of the nominal interest rate. We allow for both aggregate and idiosyncratic uncertainty and assume a lack of household insurance against idiosyncratic income risk.

## 2.1 Preferences and Technologies

**Preferences**: There is a continuum of mass 1 of infinitely lived single-member households indexed by $i \in (0,1)$. Households consume non-durable goods, $c_{i,s}$, have disutility of work, and maximize the expected discounted present value of their utility streams:

$$\mathcal{V}_{i,t} = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \frac{c_{i,s}^{1-\mu} - 1}{1-\mu} - \zeta n_{i,s} \right), \tag{1}$$

where $\mathbb{E}_t x_s = E(x_s | I_t)$ is the date $t$ conditional expectation of $x_s$, $0 < \beta < 1$ the subjective discount factor, $\mu > 0$ the measure of relative risk aversion, $n_{i,s}$ the household's employment status, and $\zeta > 0$ measures the disutility of market work. An individual household is either employed and works full-time, or does not work at all:

$$n_{i,s} = \begin{cases} 0 & \text{if not employed at date } s \\ 1 & \text{if employed at date } s \end{cases}. \tag{2}$$

The consumption level of an individual household is a CES aggregator of a basket of consumption goods, $c_i^j$:

$$c_{i,s} = \left( \int_j \left( c_{i,s}^j \right)^{1-1/\gamma} dj \right)^{1/(1-1/\gamma)}, \tag{3}$$

where $\gamma > 1$ is the elasticity of substitution between goods varieties. Workers who are not employed produce $\vartheta$ units of the aggregate consumption good at home.[5]

Households decide on consumption, savings, on the financial portfolio, and on labor force participation. A household not in the labor force cannot search for jobs in the market. Households who stand to lose on the net from employment declare themselves out of the labor force. We discuss the savings and portfolio problems later.

[5] The model would be equivalent if we assume that unemployed workers receive a constant real benefit financed by lump-sum taxation of the entrepreneurs. Assuming a constant replacement ratio instead would rule out the possibility of procyclical earnings risk. In the Appendix A9 we consider a version of the model where unemployed agents receive benefits that are sticky in nominal terms which under some conditions can provide an automatic stabilizer.

**Production technology**: Market goods are produced by a continuum of monopolistically competitive firms, indexed by $j \in (0, M)$ , that each supply a differentiated good. The technology is:

$$y_{j,s} = \exp(A_s)\, n_{j,s}, \tag{4}$$

where $y_{j,s}$ is firms $j$'s output and $n_{j,s}$ its employment. In the Appendix we consider a version of the model with physical capital. $A_s$ is an *aggregate* stochastic productivity shock which follows a first-order autoregressive process:

$$A_s = \rho_A A_{s-1} + \sigma_A \varepsilon_s^A, \tag{5}$$

where $\rho_A \in (-1, 1)$, $\sigma_A > 0$ and $\varepsilon_s^A \sim \mathcal{N}(0, 1)$.

The law of motion of employment of firm $j$ is:

$$n_{j,s} = (1 - \omega)\, n_{j,s-1} + h_{j,s}, \tag{6}$$

where $\omega$ is a constant employment separation rate and $h_{j,s}$ denotes hiring by firm $j$. Firms hire workers by posting vacancies, $v_{j,s}$, at cost $\kappa > 0$ per vacancy. A vacancy is filled with probability $q_s$. We take firms to be sufficiently large that $q_s$ is also the fraction of vacancies that are filled.[6] Thus, the total number of vacancies posted by firm $j$ is given by $h_{j,s}/q_s$.

**Matching technology**: Agents receive information about current productivity shocks at the beginning of each period. Existing worker-firm relationships are resolved at the end of the period and new ones are formed at the beginning of the next period. Households take their consumption/saving decisions after new matches are formed. Job separations are exogenous and affect existing hires randomly, so that employees perceive $\omega$ to be the risk that they lose their current job.

New hires are produced by a matching function which relates the measure of newly formed worker-firm matches to the aggregate measures of vacancies, $v_s$, and job searchers, $e_s$, as:

$$\boldsymbol{M}(e_s, v_s) = \overline{m} e_s^{\alpha} v_s^{1-\alpha}, \tag{7}$$

where $\overline{m} > 0$ indicates the match efficiency, $\alpha \in (0, 1)$, and $v_s = \int_j v_{j,s} dj$ is the measure of

[6] This is useful because we will later assume symmetry across firms and the large firm assumption avoids having to consider that the measure of vacancies filled by individual firms is stochastic.

aggregate vacancies. We impose that vacancies cannot be negative:

$$v_{js} \geq 0, \tag{8}$$

see also Petrosky-Nadeau, Zhang and Kuehn (2017). The job finding rate, $\eta_s$, i.e. the probability that a jobless worker finds a new employer, and the vacancy filling probability, $q_s$, depend on labor market tightness, $\theta_s \equiv \frac{v_s}{e_s}$, as:

$$\eta_s = \frac{\boldsymbol{M}(e_s, v_s)}{e_s} = \overline{m}\theta_s^{1-\alpha}, \tag{9}$$

$$q_s = \frac{\boldsymbol{M}(e_s, v_s)}{v_s} = \overline{m}\theta_s^{-\alpha} = \overline{m}^{\frac{1}{1-\alpha}}\eta_s^{\frac{\alpha}{\alpha-1}}. \tag{10}$$

It turns out that $\overline{m}$ and $\kappa$ enter the model equations in a way that is observationally equivalent for our purpose. Hence we impose the normalization $\overline{m} = 1$ from now on.

## 2.2 Price and Wage Setting

**Prices**: Firms set prices of their products, $P_{j,s}$, subject to a quadratic price adjustment cost as in Rotemberg (1982). The extent of nominal rigidities in price setting is parameterized by $\phi \geq 0$, which determines the size of the price adjustment costs. Let $w_s$ denote the average real wage, $y_s$ aggregate output, and $P_s$ be the aggregate price level. We anticipate that in equilibrium wages are the same for all workers and hence exclude worker- and firm specific indices for the wage. Firms maximize:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \Lambda_{j,t,s} \left[ \frac{P_{j,s}}{P_s} y_{j,s} - w_s n_{j,s} - \kappa v_{j,s} - \frac{\phi}{2} \left( \frac{P_{j,s} - P_{j,s-1}}{P_{j,s-1}} \right)^2 y_s \right], \tag{11}$$

subject to (4), (6), and a demand constraint which derives from the consumers' decision problems:

$$y_{j,s} = \left( \frac{P_{j,s}}{P_s} \right)^{-\gamma} y_s, \tag{12}$$

where $y_s = \int_j y_{j,s} dj$ denotes aggregate output and $\Lambda_{j,t,t+s}$ is the discount factor of the firm's owners (discussed in Section 2.5).

Real marginal costs, $mc_{j,s}$, are the sum of the wage and hiring costs of a marginal worker (relative to productivity). To hire a marginal additional worker at date $s$, firms must spend $\kappa/q_s$ on hiring costs but since matches persist, hiring today brings about future hiring cost savings

$(1-\omega)\,\kappa/q_s$ (discounted at the appropriate rate). Thus:

$$mc_{j,s} = \frac{1}{\exp(A_s)}\left(w_s + \frac{\kappa}{q_s} - \lambda_{v,j,s} - (1-\omega)\,\mathbb{E}_s\Lambda_{j,s,s+1}\left\{\frac{\kappa}{q_{s+1}} - \lambda_{v,j,s+1}\right\}\right), \tag{13}$$

where $\lambda_{v,j,s} \geq 0$ is the Kuhn-Tucker multiplier on (8), which satisfies the complementary slackness condition $\lambda_{v,j,s}v_{j,s} = 0$. Exploiting symmetry across firms, marginal costs equalize across firms and hence we drop the firm subscript from now on. The firms' price-setting problems deliver the following first-order condition:[7]

$$1 - \gamma + \gamma mc_s = \phi\,(\Pi_s - 1)\,\Pi_s - \phi\mathbb{E}_s\Lambda_{s,s+1}\left[\frac{y_{s+1}}{y_s}\,(\Pi_{s+1} - 1)\,\Pi_{s+1}\right]. \tag{14}$$

**Wages**: We assume that real wages are determined by Nash bargaining between workers and firms. As discussed by Krusell, Mukoyama and Sahin (2010), financial market incompleteness and risk aversion jointly imply that the surpluses that households derive from employment generally depend on their wealth levels. Hence we label the households' value and surplus functions by $i$. Firms are symmetric and hence we do not include a firm index in the bargaining equations. The wage solves the following maximization problem:

$$\max\left(S^e_{i,s}\right)^{\upsilon}\left(S^f_s\right)^{1-\upsilon}, \tag{15}$$

where $S^e_{i,s}$ is the worker's surplus, $S^f_s$ is the firm's surplus and $\upsilon \in (0,1)$ is the worker's bargaining weight. We assume that were negotiations to fall through, the worker becomes unemployed while the firm can attempt to hire a new worker in the same period. The employed worker's surplus ($S^e_{i,s}$), the difference between the value of being employed ($V^e_{i,s}$) and unemployed ($V^u_{i,s}$), is then:

$$\begin{aligned}
S^e_{i,s} &= V^e_{i,s} - V^u_{i,s},\\
V^e_{i,s} &= \frac{c^{1-\mu}_{i,e,s}}{1-\mu} - \zeta + \beta\mathbb{E}_s\omega\left(1-\eta_{s+1}\right)V^u_{i,s+1} + \beta\mathbb{E}_s\left(1-\omega\left(1-\eta_{s+1}\right)\right)V^e_{i,s+1},\\
V^u_{i,s} &= \frac{c^{1-\mu}_{i,u,s}}{1-\mu} + \beta\mathbb{E}_s\left(1-\eta_{s+1}\right)V^u_{i,s+1} + \beta\mathbb{E}_s\eta_{s+1}V^e_{i,s+1},
\end{aligned}$$

[7]Note that in the absence of price rigidities and search and matching frictions, the marginal cost equals $mc_s = \frac{w_s}{\exp(A_s)} = \frac{\gamma-1}{\gamma}$. To avoid trivial equilibria in which market work can generate no surplus to workers, even without labor market and price setting frictions, we assume that $\frac{\vartheta^{1-\mu}-1}{1-\mu} + \zeta < \frac{\left(\frac{\gamma-1}{\gamma}\exp(A_s)\right)^{1-\mu}-1}{1-\mu}$. Strictly speaking, this requires a bound on the support of the stochastic productivity process.

where $c_{i,e,s}$ ($c_{i,u,s}$) is the consumption level optimally chosen by the household in case of employment (unemployment). Recall that separations take place at the very end of the period whereas new matches are formed at the very beginning. Accordingly, the term $1-\omega\left(1-\eta_{s+1}\right)$ in the second equation is the probability that a worker employed in period $s$ is still employed in period $s+1$, either because the current match continues, or because the current breaks down but the worker immediately finds a new job in the beginning of the next period.

Since the firm will post vacancies to hire a replacement worker should the current negotiations fail, the surplus of the match to the firm satisfies:

$$S_s^f = \frac{\kappa}{q_s}. \tag{16}$$

## 2.3 Monetary Policy

The monetary authority follows an interest rate rule. Specifically, the interest rate responds to inflation, given by $\Pi_s \equiv \frac{P_s}{P_{s-1}}$, and to labor market tightness. The latter variable naturally captures, inversely, the degree of labor market slack. The interest rate rule is given by:

$$R_s = \max\left\{\overline{R}\left(\frac{\Pi_s}{\overline{\Pi}}\right)^{\delta_\pi}\left(\frac{\theta_s}{\overline{\theta}}\right)^{\delta_\theta}, 1\right\}, \tag{17}$$

where $\overline{R}$, $\overline{\Pi}$ , $\overline{\theta}$, $\delta_\pi, \delta_\theta \geq 0$ are policy parameters and the max operator captures the Zero Lower Bound (ZLB) on the net nominal interest rate, $R_s - 1$. We will later consider shocks to the monetary policy rule.

## 2.4 Financial Markets and Budget Constraints

NK models with unemployment typically assume that individual households are insured against idiosyncratic earnings shocks within large diversified families or, alternatively, that households can purchase unemployment insurance contracts at actuarially fair prices. Here we instead assume that households live in single-member families and cannot purchase unemployment insurance contracts.

Households have potentially access to two financial assets that they can invest in for self-insurance purposes. The first is a zero coupon one-period nominal bond purchased at price $1/R_s$ units of currency at date $s$. Let the household's purchases of bonds at date $s$ be given by $b_{i,s}$

(in real terms). Households must observe a liquidity constraint:

$$b_{i,s} \geq -\psi w_{i,s}, \tag{18}$$

which allows a household to borrow up to a multiple $\psi > 0$ of its current wage income.

A second asset that is available to households is firm equity which are claims to the dividend streams of the firms. Let $x_{i,s}$ denote household $i$'s purchases of equity in period $s$. We impose the following constraint:

$$x_{i,s} \geq 0, \tag{19}$$

which rules out the option to go short in equity.

The households choose their consumption streams and make their savings and portfolio choices subject to a sequence of budget constraints:

$$c_{i,s} + \frac{b_{i,s}}{R_s} + p_{x,s}x_{i,s} \leq w_{i,s}n_{i,s} + \vartheta\left(1 - n_{is}\right) + \frac{b_{i,s-1}}{\Pi_s}b_{i,s-1} + \left(1 - \tau_i\right)\left(p_{x,s} + d_{x,s}\right)x_{i,s-1},$$

where $p_{x,s}$ denotes the equity price and $d_{x,s}$ is the dividend payment. Consistently with the empirical evidence of Fagereng et al (2016), this formulation allows for persistent differences across households in asset returns through the parameter $\tau_i \in [0, 1]$.

## 2.5 Conditions for a Tractable Equilibrium

Without further assumptions, the model can only be solved numerically. In this paper we aim at an analytical characterization of steady-state equilibria, as well as of the local fluctuations around those steady states. To achieve this, we first impose that $\tau_i = \{0, 1\}$. That is, a subset of the households is unable to obtain returns from equity investment and hence is effectively blocked from participation in the equity market. We denote the fraction of households who are able to invest in equity by $\xi$ and we assume that $\tau_i = 0$ for $i < \xi$ and $\tau_i = 1$ for $i \geq \xi$. We will assume that $\xi$ is small, in a sense that will be detailed below. Second, we assume that the net aggregate supply of bonds is zero.

While these assumptions might seem fairly minor, they simplify the analysis considerably

since they lead to equilibria in which the following two conditions hold:

$$b_{i,s} = 0 \ \forall \ i, \tag{20}$$

$$1 = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left( \omega \left(1 - \eta_{s+1}\right) \left( \frac{\vartheta}{w_s} \right)^{-\mu} + \left(1 - \omega \left(1 - \eta_{s+1}\right)\right) \left( \frac{w_{s+1}}{w_s} \right)^{-\mu} \right)^{-1}. \tag{21}$$

The first of these conditions states that all individual households hold exactly zero bonds. The second equation is an asset-pricing condition, which relates the real interest rate to real wages and the job finding rate. As we will show below, this is the Euler equation of the employed households. Importantly, the equation contains only aggregate variables, which will allow us to solve for the equilibrium real interest rate without reference to the distribution of wealth.

We now verify that (20) and (21) are indeed consistent with utility maximization of all households, and with market clearing. A direct consequence of Equation (20) is that all households consume their current income. We can therefore distinguish between three distinct groups of households, with consumption levels given by:

$$c_{i<\xi,u,s} = \frac{1}{\xi} \left( y_s - \kappa v_s - w_s n_s - \frac{\phi}{2} \left( \Pi_s - 1 \right)^2 y_s \right) + \vartheta, \tag{22}$$

$$c_{i \geq \xi,u,s} = \vartheta, \tag{23}$$

$$c_{i \geq \xi,e,s} = w_s. \tag{24}$$

Equation (22) is the consumption of those who can invest in equity. The first term on the right hand side is the dividend that an individual equity holder receives from the firms. Since firms make monopoly profits, the steady-state dividend is typically positive. These profits are spread out over the fraction of households who hold equity, $\xi$. The second term is the home production. Here we have assumed that $\xi$ is small enough for the equity investors to have enough dividend income to drop out of the labor market. This assumption simplifies the analysis since now the equity investors are no longer exposed to idiosyncratic risk.[8]

Equation (23) is the consumption of the asset-poor employed households, whereas Equation (24) is the consumption of the asset-poor unemployed households. Since there is no heterogeneity across households conditional on their type and employment status, we drop the $i$-subscript and denote consumption levels as $c_{e,s} = c_{i \geq \xi,e,s}$, $c_{u,s} = c_{i \geq \xi,u,s}$, and $c_{r,s} = c_{i<\xi,u,s}$, where subscript $r$ denotes the asset-rich households. Note also that firms now discount profits at a common rate

[8] Even if these households were to participate in the labor market, they would be relatively well insured against idiosyncratic risk due to wealth (equity holdings).

$\Lambda_{t,s} = \beta \left(c_{r,t}/c_{r,s}\right)^{\mu}$ since they are owned by the asset-rich households who face only aggregate risk.

The above outcomes are trivially consistent with clearing of the goods market and the bond market. To see why they also satisfy optimality of households decisions, consider in turn the Euler equations for bonds for the asset rich, the unemployed and the employed:

$$c_{r,s}^{-\mu} \geq \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{r,s+1}^{-\mu}, \tag{25}$$

$$c_{u,s}^{-\mu} \geq \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left( \left(1-\eta_{s+1}\right) c_{u,s+1}^{-\mu} + \eta_{s+1} c_{e,s+1}^{-\mu} \right), \tag{26}$$

$$c_{e,s}^{-\mu} \geq \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left( \omega \left(1-\eta_{s+1}\right) c_{u,s+1}^{-\mu} + \left(1-\omega\left(1-\eta_{s+1}\right)\right) c_{e,s+1}^{-\mu} \right), \tag{27}$$

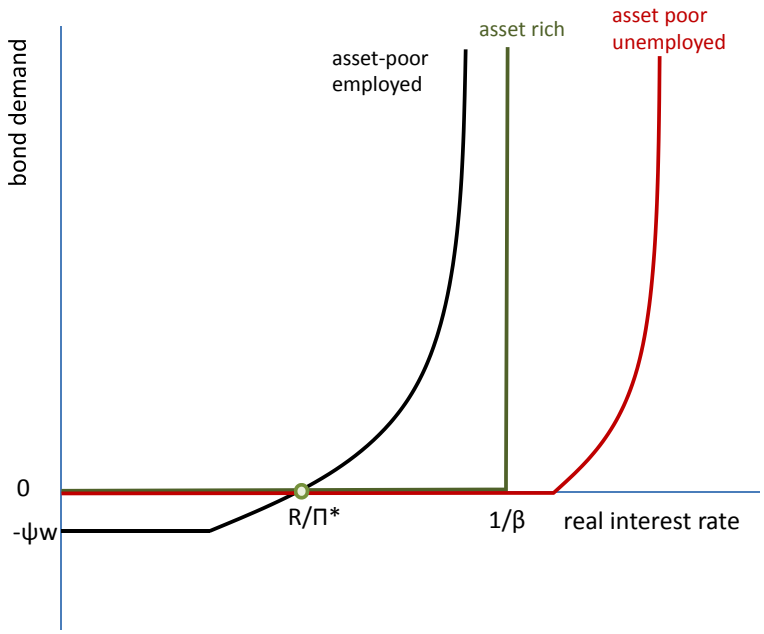
where each condition holds with equality if the household is not liquidity constrained and with strict inequality when the liquidity constraint binds. The conjecture implies that in any steady state the real interest rate lies below the subjective discount rate, i.e. $\frac{R}{\Pi} < \frac{1}{\beta}$. Both (25) and (27) are consistent with this provided that they do not hold with equality.[9] The Euler Equation of the employed households is also consistent with the conjecture. This can be seen by combining (27) with (23) and (24), which induces (21), the Euler equation holds with equality. The latter is necessary since otherwise the employed households would face a binding borrowing constraint. This, however, would mean that they hold positive amounts of debt, which would violate bond market clearing given that the asset-rich and the unemployed hold zero bonds.[10]

To see more intuitively why this equilibrium arises, consider Figure 2, which depicts the steady-state bond demand schedules of the three groups of households, as functions of the real interest rate. Generally these functions are upward sloping. To the far right is the demand schedule of the unemployed. These households have a strong incentive to borrow, realizing that in the future they might find a job and receive more income. In the middle is the bond demand function of the asset rich. In order for these households to be willing to hold any bonds in the

[9] To see this note that in a steady state (25) reduces to $\frac{R_s}{\Pi} \leq \frac{1}{\beta}$, whereas (27) reduces to $\frac{R_s}{\Pi} \leq \frac{1}{\beta}((1-\eta) + \eta \left(\frac{\vartheta}{w}\right)^{-\mu})^{-1} < \frac{1}{\beta}$.

[10] One might wonder if there are related steady-state equilibria in which the asset rich and the unemployment are at the constraint, but there is heterogeneity in wealth and consumption within the group of employed households. This is not the case. To understand why, first note that on average the employed would have to hold zero bonds. Second, note that any employed household just coming out of unemployment would start with zero wealth. At the conjectured interest rate, they choose to buy exactly zero bonds. However, if the real interest rate were higher than the one conjectured, households would gradually accumulate bonds during the employment spell, monotonically converging to a certain target level of wealth. This, however, would mean that average bond holdings are positive, violating bond market clearing. Conversely, if the interest rate were lower than the one conjectured, aggregate bond holdings would be negative.

Figure 2: Illustration of steady-state bond demand schedules.




steady state, it must be that the real interest rate equals at least $\frac{1}{\beta}$. Below that, strictly firm equity dominates bonds in return and hence the equity holders would like to borrow in order to buy more equity. However, the liquidity constraint prevents them from doing so. The left schedule represents the employed, asset-poor households, who are most eager to save, due to their precautionary savings motive. They end up holding zero bonds, but not because they are forced by a constraint. Indeed, they are away from their constraint and could in principle save or borrow. Rather, the equilibrium real interest rate adjusts downward to a point at which they voluntarily hold zero bonds. This is required for clearing of the bond market, given that the asset rich and the unemployed hold zero bonds due to the liquidity constraint, which binds for them.

We thus arrive at a tractable steady state in which all households hold zero bonds and the asset-poor employed households determine the real interest rate, and the remaining households face a binding liquidity constraint. Importantly, individual employed households could in principle borrow or lend as they see fit, since they are away from their liquidity constraint. However, they choose not to do so since in equilibrium the real interest rate adjusts to a point at which they wish to hold zero bonds, which is required for bond market clearing. The fact that we allow those who price the bond to borrow sets us apart from earlier literature achieving tractability in incomplete-markets models, which typically assumes that no one can borrow, see Krusell,

Mukoyama and Smith (2011), Werning (2015), McKay, Nakamura and Steinsson (2017), Mckay and Reis (2017), Bilbiie (2017) and Ravn and Sterk (2017).[11]

Now consider local equilibria around the steady state, driven by aggregate shocks. Provided that these shocks are not too large, the asset rich and unemployed are liquidity constrained, whereas the employed are not. Since there is no aggregate supply of bonds in which the employed can save, the employed perpetually choose to hold zero bonds. Thus, (20) and (21) continue to hold in the neighborhoods of steady states.[12]

Finally, consider the equilibrium labor market flows. Provided that the asset-poor are unwilling to leave the labor force, the labor market participation rate is constant and given by $1-\xi$. In that case, the aggregate unemployment rate is given by:

$$u_s = 1 - n_s, \tag{28}$$

where $n_s = \frac{1}{1-\xi}\int_j n_{j,s}dj$ is the aggregate employment rate, as a fraction of the labor force. The law of motion of unemployment is given as:

$$u_s = u_{s-1} + \omega n_{s-1} - h_s, \tag{29}$$

where $h_s = \frac{1}{1-\xi}\int_j h_{j,s}dj$ is the number of new hires as a fraction of the labor force. The aggregate number of job searchers is given by $e_s = (1-\xi)\left(u_{s-1} + \omega n_{s-1}\right)$.

## 2.6 The Endogenous Risk Channel

Before we move on to characterize formally the equilibrium outcomes, it is useful to outline the essential mechanism at play in the model. There are four key relationships in the model:

$$\begin{aligned}
c_{e,s}^{-\mu} &= \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{e,s+1}^{-\mu}\left[1+\omega\left(1-\eta_{s+1}\right)\left(\left(c_{u,s+1}/c_{e,s+1}\right)^{-\mu}-1\right)\right], \\
1-\gamma+\gamma mc_s &= \phi\left(\Pi_s-1\right)\Pi_s - \phi \mathbb{E}_s \Lambda_{s,s+1}\left(\Pi_{s+1}-1\right)\Pi_{s+1} y_{s+1}/y_s, \\
mc_{j,s} &= \frac{1}{\exp(A_s)}(w_s + \kappa \eta_s^{\frac{-\alpha}{\alpha-1}} - (1-\omega)\,\mathbb{E}_s \Lambda_{j,s,s+1}\kappa \eta_{s+1}^{\frac{-\alpha}{\alpha-1}}), \\
R_s &= \overline{R}\left(\frac{\Pi_s}{\overline{\Pi}}\right)^{\delta_\pi}\left(\frac{\theta_s}{\overline{\theta}}\right)^{\delta_\theta}.
\end{aligned}$$

[11] In those studies, there is typically a continuum of equilibrium real interest rates, which is not the case in our analysis.

[12] It is worth stressing that the property that consumption equals income for the households is an equilibrium feature. Since we allow for debt, the marginal propensity to consume out of an idiosyncratic temporary income shock would not equal one in our model.

The first of these is the Euler equation of the employed workers, which we discuss further below. The second condition is the optimal price-setting equation, which relates inflation to real marginal costs as in the standard NK model. The third equation defines real marginal costs as a function of real wages and hiring costs, which in turn depend on the job finding rate. The last equation is the interest rate rule. We have for simplicity ignored the non-negativity constraint on vacancies and the zero lower bound on the nominal interest rate.

Apart from the first equation, the other expressions are identical to those in a standard three-equation NK model (obtained by combining the second and third equations). The modification of the first equation, the Euler equation of the employed households (21), is key. The modification relates to the term between square brackets which is a wedge that arises due to market incompleteness. This term captures the precautionary savings motive of the employed households generated by unemployment risk. The wedge collapses to one if either there is no risk of unemployment ($\omega = 0$ or $\eta = 1$), if markets are effectively complete ($c_{u,s+1} = c_{e,s+1}$), or households are risk neutral ($\mu = 0$). In each of these cases, the Euler equation reduces to $c_{e,s}^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{e,s+1}^{-\mu}$, coinciding precisely with the standard representative-agent Euler equation. In that case, the model the model has no interesting aggregate implications beyond the standard NK model.

When these conditions are not satisfied, i.e. when income risk is imperfectly insured and agents are risk averse, the wedge is strictly greater than one and varies with labor market conditions. The properties of the model in this case will depend crucially on the cyclical nature of the incomplete markets wedge and on the endogeneity of the wedge.

Consider first the case when earnings risk is exogenous. Assume, for example, that the labor market transition rates are exogenous and denote $p_s^{eu}$ as the exogenous probability that an employed worker is unemployed next period.[13] For simplicity let the drop in consumption (and income) upon job loss be exogenous and given by $\vartheta/w_s$. We can then express the Euler equation as:

$$c_{e,s}^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{e,s+1}^{-\mu} \left[1 + p_s^{eu} \left( (\vartheta / w_{s+1})^{-\mu} - 1 \right) \right].$$

This version of the model with exogenous earnings risk still adds a precautionary savings term to the model which impacts negatively on demand when $p_s^{eu}$ rises or when $\vartheta/w_{s+1}$ falls. In both cases, goods demand from employed households is depressed due to a precautionary savings

[13] One could modify the model explicitly to arrive at this version, by assuming that separated workers can only search for jobs with a one-period delay. In that case, $p_s^{eu}$ equals the job separation rate, which one could make (exogenously) time-varying.

motive as the risk of unemployment or the costs thereof rises. This demand contraction lowers activity due to firms adjusting prices slowly. However, the supply side adjustment has no further effects on the demand side of the economy and this version of the model has limited aggregate implications.

Our main interest is instead in the case where the incomplete markets wedge is *endogenous* as is the case in our model due to frictional unemployment as well as the wage being endogenous. In this case, the model introduces a feedback mechanism between the supply side and the demand side of the economy which induces either amplification or stabilization, depending on structural parameters. Consider a drop in the job finding rate. Lower job finding rates imply increased idiosyncratic earnings risk for employed households because they realize that new jobs are harder to find should they lose their current position. This motivates an increase in desired precautionary saving. On the other hand, a lower job finding rate also tends to decrease the real wage, since the outside opportunities of workers have deteriorated. Lower real wages, in turn, imply a smaller drop in income upon job loss, which discourages precautionary saving.

When the first of these effects dominates, a case we refer to as *countercyclical* endogenous earnings risk, deteriorating labor market conditions induce employed agents to lower their goods demand at the current real interest rate, which puts downward pressure on the real interest rate. As long as the central bank operates an active rule for the nominal interest rate, the downward pressure on the real interest rate induces a drop in the inflation rate. According to the condition for optimal price setting, lower inflation requires real marginal costs to fall. Lower marginal costs, in turn, requires either real wages to drop or hiring costs to decline. In general, both real wages and hiring costs decline. The latter, in turn, requires firms to hire less, which induces a further decline in the job finding rate. This sets in motion a further reduction in demand. This feedback mechanism will operate to amplify fluctuations in the economy. Conversely, when the wage effect dominates, the procyclical endogenous earnings risk case, the wedge has a stabilizing effect because the demand for precautionary savings increases in booms and declines in recessions. Hence, the properties of the endogenous risk that arise in the model will be key for the model's implications.

Because the cyclicality of risk is central to the discussion that follows, it is useful to consider whether the endogenous risk wedge is likely to be procyclical or countercyclical. Storesletten, Telmer and Yaron (2004) study PSID household income data and conclude that idiosyncratic labor market income risk is strongly countercyclical. More recently, studying a 10 percent sample of all U.S. working-age males , Guvenen, Ozkan and Song (2014) show that the countercyclicality

of earings risk derives from increased left-skewness in recessions (i.e. higher likelihood of large earnings losses and lower likelihood of large gains) rather than from a countercylical variance.

These findings indicate that sources of countercyclical risk are likely to dominate. Furthermore, our emphasis on unemployment risk is consistent with the left-skewness of earnings changes in recessions as emphasized by Guvenen, Ozkan and Song (2014). In Section 7, we will use the model to further argue that the countercylical risk is the empirically relevant case, once one takes a view on plausible values of the parameters entering the wedge. Moreover, countercyclical risk allows the model to match the relationship between the real interest rate and labor market tightness, as illustrated in Figure 1 above. For these reasons, we concentrate most of our discussion on the case where the endogenous earnings risk wedge is countercyclical, although we will point out how this contrasts with the procyclical case.

# 3 Steady-state Equilibria

This section discusses the set of steady-state equilibria that can arise absent aggregate shocks.

## 3.1 Global Determinacy

Consider a version of the model without any aggregate shocks. An important difference vis-à-vis the extant complete-markets NK literature is that although aggregate variables are constant in the steady state, the labor market participants still face idiosyncratic risk due to lack of insurance against earnings risk.

We indicate steady-state values by removing time subscripts from variables. Define for convenience $R^* \equiv \overline{R}\,\overline{\Pi}^{-\delta_\pi}\overline{\theta}^{-\delta_\theta}$. The solution for the steady-state wage can be expressed as function of the job finding rate, $\boldsymbol{w}(\eta)$. This function is derived in Appendix A1, which also contains a characterization of some of its basic properties. Steady-state equilibria can be characterized by the solutions to:

$$\phi(1-\beta)(\Pi-1)\Pi = 1-\gamma+\gamma\left(\boldsymbol{w}(\eta)+\left(\kappa\eta^{\alpha/(1-\alpha)}-\lambda_v\right)(1-\beta(1-\omega))\right), \qquad (\mathbf{PC})$$

$$1 = \beta\frac{\max\{R^*\Pi^{\delta_\pi}\eta^{\delta_\theta/(1-\alpha)},1\}}{\Pi}\boldsymbol{\Theta}^{SS}(\eta), \qquad (\mathbf{EE})$$

where $\lambda_v \geq 0$ (the Kuhn-Tucker multiplier on the vacancy constraint (8)), $\lambda_v v = 0$, and where $\boldsymbol{\Theta}^{SS}(\eta)$ is the steady-state endogenous risk wedge, which can be expressed as a function of the

job finding rate:

$$\boldsymbol{\Theta}^{SS}(\eta) \equiv 1 + \omega(1-\eta)\left[(\vartheta/\boldsymbol{w}(\eta))^{-\mu} - 1\right] \geq 1.$$

(**PC**) and (**EE**) can both be considered as defining a relationship between the job finding rate $\eta$ and the inflation rate $\Pi$ and the steady-state equilibria relate to the intersections of these relationship. Equation (**PC**) is the steady-state version of (14), the optimality condition for price setting ("the Phillips Curve"). It generally defines a positive relationship between inflation, $\Pi$, and the job finding rate, $\eta$. Intuitively, high job finding rates imply low vacancy filling rates and higher wages, as the competition for workers intensifies. This drives up marginal costs, and hence encourages firms to increase prices. The left-hand side of the equation is a standard steady-state sticky-price wedge, which vanishes in the absence of price adjustment costs ($\phi = 0$). In that case, the job finding rate is determined independently from the rate of inflation. A similar disconnect occurs when the non-negativity constraint on vacancies binds ($\lambda_v > 0$) and hiring freezes ($\eta = 0$).

Equation (**EE**) is the steady-state version of the employed households' Euler equation (21). This equation also defines a relation between $\Pi$ and $\eta$ (consistent with employed agents maximizing utility). The slope of this schedule is determined by the impact of the job finding rate on the risk wedge and by whether the net nominal interest rate is positive or not. $\boldsymbol{\Theta}^{SS}(\eta)$ may be increasing or decreasing in $\eta$, which in turn depends on the cyclicality of earnings risk. When $\boldsymbol{w}'(\eta) \simeq 0$ so that wages are unresponsive to the job finding rate, earnings risk is countercyclical and $\partial\boldsymbol{\Theta}^{SS}(\eta)/\partial\eta < 0$ because unemployment risk dominates.[14] When wages are sufficiently elastic, i.e. when $\boldsymbol{w}'(\eta)$ is large, overall earnings risk becomes procyclical, so that $\partial\boldsymbol{\Theta}^{SS}(\eta)/\partial\eta \geq 0$. In this case, wages increase sharply with the job finding rate implying a large income loss in a high job finding rate steady-state.

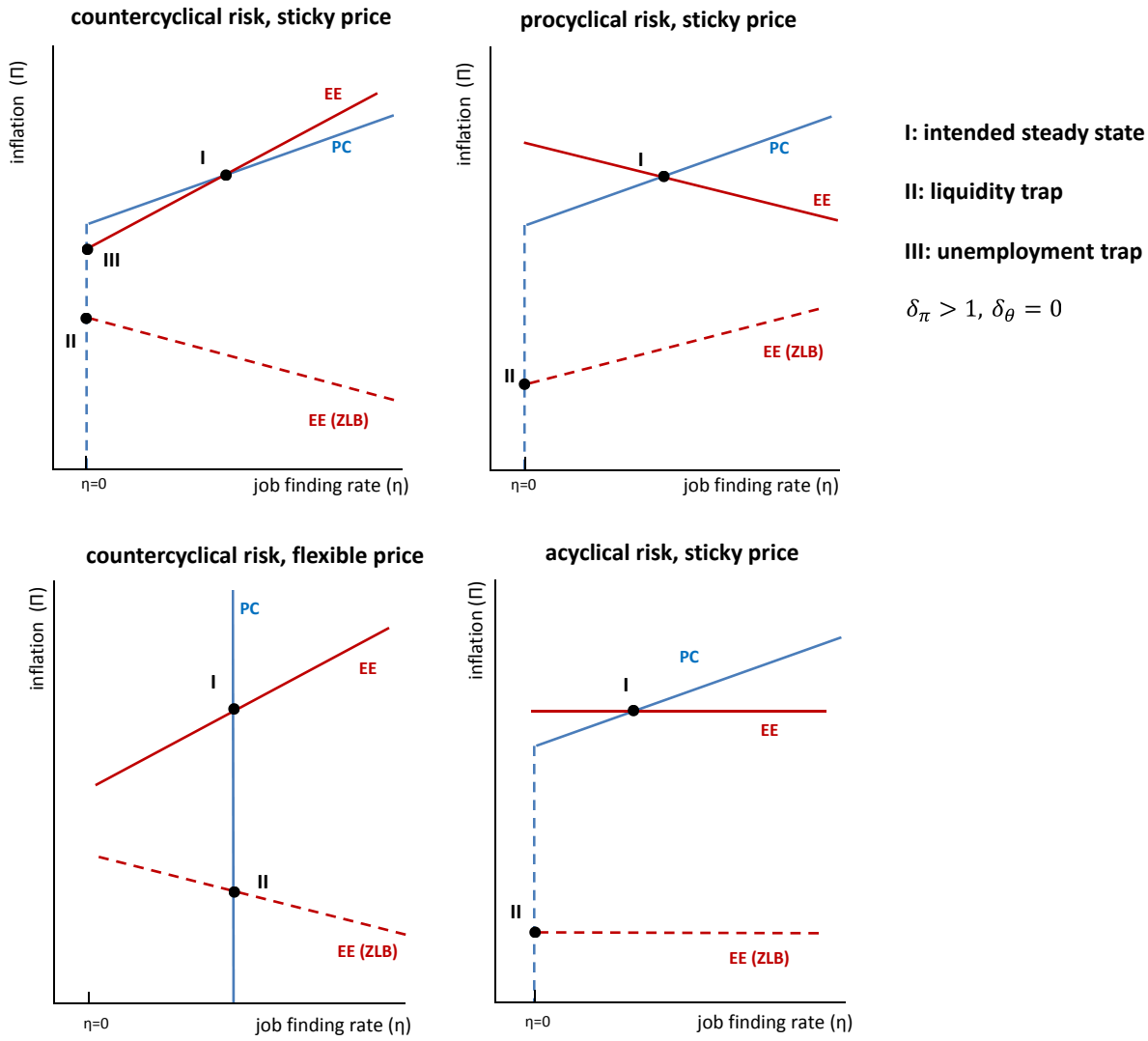
Second, the slope of the **EE** schedule depends on whether or not the ZLB on the nominal interest rate binds. For simplicity, consider initially a case in which the interest rate rule only reacts to inflation (i.e. $\delta_\theta = 0$) and satisfies the Taylor principle ($\delta_\pi > 1$), so that away from the ZLB the real interest rate is increasing in the inflation rate. While the slope of **EE** schedule depends on $\boldsymbol{\Theta}^{SS}(\eta)$, the sign of the slope reverses under a binding ZLB.[15]

Figure 3 illustrates the steady-state schedules. Under countercyclical risk, the **EE** schedule is positively sloped when the ZLB does not bind. Intuitively, when the job finding rate is high,

[14] Formally, $\partial\boldsymbol{\Theta}^{SS}(\eta)/\partial\eta = -\omega[(\vartheta/w)^{-\mu} - 1] + \mu\omega(1-\eta)/\eta(\vartheta/w)^{-\mu}\chi$ where $\chi = (\partial w/\partial\eta)(\eta/w)$ is the elasticity of the real wage to the job finding rate.

[15] To see this, note that, away from the ZLB, the real interest rate is given by $R^r = \Pi^{\delta_\pi - 1}$, so that $\frac{\partial R^r}{\partial \Pi} > 0$, given $\delta_\pi > 1$. At the ZLB, the real interest rate equals $R^r = 1/\Pi$, which implies $\frac{\partial R^r}{\partial \Pi} < 0$.

Figure 3: Illustration of steady-state equilibria.




the precautionary savings motive is weak, implying a relatively high real interest rate. Since monetary policy responds more than one-for-one to inflation, a high real interest rate implies a high rate of inflation. At the ZLB the schedule, denoted **EE**(ZLB), slopes downward. The exact opposite is true under procyclical risk, i.e. **EE** slopes downward away from the ZLB whereas **EE**(ZLB) slopes upward. Under acyclical risk the **EE** schedule is horizontal, both at the ZLB and away from it.

Consider now the upper left panel of Figure 3, which illustrates a baseline case with countercyclical risk and sticky prices. Three possible steady states emerge:[16]

[16]We ignore the possibility of an additional equilibrium that occurs due to the quadratic price adjustment term, or due to non-linearities in $\boldsymbol{w}(\eta)$.

I *Intended steady state.* This steady state occurs at the intersection of the **PC** and the **EE** schedule at $\eta > 0$. This is the "intended" steady state, at which the ZLB does not bind and the job finding rate is relatively high.

II *Liquidity trap.* This steady state arises because of the ZLB on the nominal interest rate and occurs at the intersection of the **PC** and the **EE**(ZLB) schedule. This is the "liquidity trap" examined by Benhabib, Schmitt-Grohé and Uribe (2001, 2002) and Mertens and Ravn (2014). This steady state features a lower rate of inflation than the intended steady state, as well as a lower job finding rate. In fact, the job finding rate is zero in the illustration.

III *Unemployment trap.* This steady state occurs at the intersection of the **PC** the **EE** schedule at $\eta = 0$. In this equilibrium, investment in vacancies comes to a complete standstill, despite the fact that the ZLB on the nominal interest rate does not bind. Note that the inflation rate in this steady state lies in between those in the intended steady state and the liquidity trap.[17]

The first two of these types of equilibria occur also when the endogenous risk is procyclical and in standard complete-markets representative-agent NK models. There are, however, important differences between the properties of the equilibria under complete and incomplete markets, coupled with endogenous earnings risk. Under complete markets, the steady-state real interest rate needs to equal $1/\beta$ in order to be consistent with constant consumption. Without full insurance, and regardless of the slope of $\mathbf{\Theta}^{SS}(\eta)$, the wedge in (**EE**) exceeds unity, which reduces the equilibrium real interest rate below the inverse of the discount factor, $\frac{R}{\Pi} < \frac{1}{\beta}$ because of the precautionary savings motive induced by unemployment risk. Therefore, economic policy is a co-determinant of the long-run real interest rate. As long as equilibrium wages do not depend on market incompleteness, however, the central bank can replicate the steady-state levels of unemployment and inflation that would prevail under complete markets. Suppose for example that the central bank targets price stability, $\overline{\Pi} = 1$, and let $\eta^{CM}$ denote the steady-state job finding rate under complete markets. Then the central bank can implement the same outcome under incomplete markets by setting $\overline{\theta} = \left(\eta^{CM}\right)^{\frac{1}{1-\alpha}}$ and $\overline{R} = 1/\left(\beta \mathbf{\Theta}^{SS}\left(\eta^{CM}\right)\right)$. This is not possible, however, without the use of fiscal policy if wages depend on market incompleteness, as

[17] To avoid the labor market totally collapsing one can e.g. introduce the possibility that some jobs are filled without the need to post vacancies, which would lead to a strictly positive lower bound on the job finding rate.

they will in general.[18]

**The Unemployment Trap**: The possible emergence of a third steady state is perhaps the most interesting consequence of introducing incomplete markets. The existence of this steady state depends critically on the interaction between countercyclical risk and sticky prices. If prices are flexible, or the endogenous earnings risk is acyclical or procyclical, this steady state cannot exist, see Figure 3. Thus, it is the interaction between sticky prices and endogenous risk that paves the way for this bad equilibrium.

Intuitively, the unemployment trap steady state arises when endogenous risk is sufficiently countercyclical, so that expectations of poor labor market conditions trigger such an increase in desired savings that firms cut sufficiently much back on hiring that the economy spirals towards an outcome where firms no longer want to hire because of lack of demand for their goods. For this to be possible, endogenous risk must be sufficiently countercyclical that the Euler equation schedule becomes steeper than the Phillips curve schedule.

The likelihood of the existence of the unemployment trap is higher when monetary policy reacts little to inflation and/or labor market tightness, and hiring costs are limited.[19] If the central bank implements sufficiently aggressive policies (policy rules with sufficiently large values of $\delta_\pi$ and/or $\delta_\theta$), the unemployment trap can be ruled out because the central bank neutralizes the impact of deteriorating labor market conditions through interest rate cuts. We discuss the details of this in the Appendix.

In the unemployment trap, agents will in the limit survive through the availability of home-production. One might consider this outcome too extreme but it is straightforward to extend the model so that the unemployment trap displays high but finite unemployment.[20] Consider the following alternative specification of the matching function

$$\boldsymbol{M}(e_s, v_s) = \overline{m} e_s^{\alpha} v_s^{1-\alpha}, \tag{30}$$

where $v_s = \int_j (v_{j,s} + \widetilde{v})\, dj$ is the measure of aggregate vacancies. Here $v_{j,s}$ denotes the mea-

[18] The reason is that wages impact on the Phillips curve as well. In this case, the intended steady state can replicated by taxing labor income and by adjusting $\overline{R}$.

[19] One intriguing issue is that the unemployment trap can be ruled out if the government allows the real interest rate to depend negatively on inflation. Such a policy, however, would make the intended steady-state locally indeterminate.

[20] Interestingly, when the central bank's policy rule for the short term interest rate depends on labor market tightness, the unemployment trap, if it exists, displays low but strictly positive job finding rates. We discuss this in Appendix A6.

sure of formal vacancies post by firm j (at the flow cost $\kappa$ per vacancy) while $\widetilde{v} \geq 0$ denotes a fixed amount of "informal" vacancies, which are costless. The idea here is that some job matches are formed even when firms do not actively post any vacancies. This happens through informal channels such as "word-of-mouth." There is empirical support for such informal channels; Davis, Faberman and Haltiwanger (2013), for example, show that a substantial fraction of establishment-level hiring takes place without formal vacancies having been posted.

For this specification of the matching function, the economy still displays an unemployment trap where firms stop posting formal vacancies but due to the availability of "informal vacancies," market activity does not get wiped out in the limit and the job finding rate in the unemployment trap is strictly positive and given as:

$$\widetilde{\eta} = \frac{\boldsymbol{M}(\widetilde{e}, \widetilde{v})}{\widetilde{e}},$$

where $\widetilde{e}$ satisfies the condition:

$$\widetilde{e} = (1-\xi)\left(\frac{\omega\,(1-\widetilde{\eta})}{\omega\,(1-\widetilde{\eta})+\widetilde{\eta}}\,(1-\omega)+\omega\right).$$

In Appendix A6 we illustrate graphically the determination of the unemployment trap in this version of the model. The outcome is equivalent to the situation in Figure 3, apart from the limit point displaying a low but positive job finding rate.

The unemployment trap is an intriguing outcome. The slow recovery after the Great Recession and the very protracted nature of the surge in unemployment observed in the U.S. (and many other OECD economies) have spurred a renewed interest in "secular stagnation," equilibrium outcomes consistent with long periods of low activity and high unemployment. Hansen (1939) argued that such outcomes (with negative natural real interest rates) were most likely produced by a combination of low rate of technological progress and population ageing, implying high savings rates and low investment rates. Recently, Eggertsson and Mehrotra (2014) have argued that deleveraging may lead to secular stagnation and exacerbate the problems that follow from an ageing population and falling investment goods prices.

The unemployment trap that can arise in our model offers an alternative perspective of secular stagnation, which ties together low real interest rates, high unemployment and low activity. Importantly, the unemployment trap can occur in our model purely because of expectations and thus does not rely on sudden changes in population growth, technological progress or financial tightening. Furthermore, while the nominal interest rate may be low in the unemployment trap,

its root cause does not derive from the ZLB on nominal interest rates. Therefore, the ongoing discussions about re-design of monetary policy to prevent secular stagnation by avoiding the ZLB may be in vain.

## 3.2 Local Determinacy

**The log-linearized model:** We now log-linearize the model in order to study the local stability properties of the equilibria. Let a hat denote a log deviation from the intended steady state, i.e. $\widehat{x}_s = \ln x_s - \ln \overline{x}^I$, where $\overline{x}^I$ denotes the value of $x_s$ in the intended steady state (discussed above). We assume that monetary policy parameters are such that $\overline{R}$, $\overline{\theta}$ and $\overline{\Pi}$ correspond to the levels of, respectively, $R$, $\theta$ and $\Pi$, in the intended steady state.

The log-linearized Euler equation of the employed households, (21), can be expressed as (see Appendix A2 for details):

$$\begin{aligned} -\mu\widehat{c}_{e,s} + \mu\beta\overline{R}\mathbb{E}_s\widehat{c}_{e,s+1} &= \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1} - \underbrace{\beta\overline{R}\Theta^F\mathbb{E}_s\widehat{\eta}_{s+1}}_{\text{endogenous risk wedge}}, \\ \Theta^F &\equiv \omega\eta\left((\vartheta/w)^{-\mu} - 1\right) - \chi\mu\omega\left(1-\eta\right). \end{aligned} \tag{31}$$

where $\chi$ is a parameter that measures the elasticity of real wages to the job finding rate, see below.

$\widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1}$ is the real interest rate while the last term on the right-hand side is the endogenous risk wedge, which fluctuates proportionally with the expected job finding rate and captures the precautionary savings motive.[21] Its strength and cyclicality is determined by $\Theta^F$, which depends on structural parameters. The first part, $\omega\eta\left((\vartheta/w)^{-\mu} - 1\right) > 0$, captures the impact of earnings risk due to fluctuations unemployment risk which, as above, creates a tendency for countercyclical endogenous earnings risk.[22] The second part, $-\chi\mu\omega\left(1-\eta\right) < 0$, relates to changes in earnings risk which derive from wage fluctuations which is a procyclical risk channel as long $\chi > 0$. The sign of $\Theta^F$ depends on which of these two channels dominates.

If consumption losses upon unemployment are large, e.g. in the face of little insurance, the first of these sources will tend to dominate. On the other hand, if wages are elastic and procyclical wages (high $\chi$), the second channel is more likely to dominate. We refer to $\Theta^F > 0$ as

[21] The incomplete markets wedge that occurs in the log-linearized Euler equation differs from its steady state version because of a different normalization and because of the impact of wage fluctuations on savings, see below.

[22] The first part of $\Theta^F$ becomes zero if the steady-state job finding rate, $\eta$, equals zero. The reason for this is technical, however. Note that $\mathbb{E}_s\widehat{\eta}_{s+1}$ is the *percentage* deviation in the expected job finding rate from its steady-state value. If the steady-state value is zero, no percentage deviation represents any actual change.

countercyclical endogenous earning risk and $\Theta^F < 0$ as procyclical earnings risk. When $\Theta^F = 0$, the endogenous risk wedge vanishes and the above equation reduces to the log-linearized Euler equation obtained in standard representative-agent models, aside from a weight $\beta\overline{R} < 1$ on next period's consumption.[23] As we have argued earlier, the countercyclical risk case is likely to be the more empirically relevant case, so we will concentrate our discussion of the results around this case.

Next, we log-linearize the firms' price-setting condition, Equation (14), around the intended steady state:

$$\frac{\phi}{\gamma}\widehat{\Pi}_s - \beta\frac{\phi}{\gamma}\mathbb{E}_s\widehat{\Pi}_{s+1} = w\widehat{w}_s + \frac{1-\gamma}{\gamma}A_s + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \beta(1-\omega)\frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1} + \mathbb{E}_s\widehat{\Lambda}_{v,s+1}\right), \quad (32)$$

where we have exploited that $q_s = \eta_s^{-\frac{\alpha}{1-\alpha}}$. For now, we abstract from productivity shocks, setting $A_s = A_{s+1} = 0$ at any date $s$. The left-hand side of the above equation is the sticky-price wedge, which vanishes in the absence of nominal rigidities ($\phi = 0$) or in the limit with perfect competition ($\gamma \to \infty$). The right-hand side is the log-linearized marginal cost, which is standard given the presence of search and matching frictions.

The log-linearized policy rule reads:

$$\widehat{R}_s = \delta_\pi\widehat{\Pi}_s + \delta_\theta\widehat{\theta}_s. \quad (33)$$

In Appendix A2, we further show that the log-linearized bargaining equations imply that:

$$\widehat{w}_s = \chi\widehat{\eta}_s, \quad (34)$$

where $\chi$ is a convolution of the model's deep parameters, which captures the sensitivity of the wage to fluctuations the job finding rate and depends critically on the bargaining parameter $\upsilon$. Finally, note that in equilibrium the employed households consume their wage, i.e. $\widehat{c}_{e,s} = \widehat{w}_s$.

**Reducing the model to a single equation:** For maximal tractability, we introduce two further assumptions which allow us to reduce the model to a single equation. First, we set the monetary policy coefficient equal to $\delta_\pi = \frac{1}{\beta} > 1$. This is inconsequential, since the coefficient on tightness, $\delta_\theta$, is left unrestricted.[24] Second, we assume that the households who can invest in

[23] This weight is not unimportant, however. McKay, Nakamura and Steinsson (2017) show that it is instrumental in alleviating the "forward guiding puzzle".

[24] The log-linearized model contains no endogenous state variables and hence for any desire pair of values $\delta_\pi$

equity (i.e. those with index $i < \xi$) are risk neutral. In this case, the log-linearized model has no endogenous state variables. In Appendix A3, we relax this assumption. The results suggest that allowing for risk-averse equity investors has only very limited implications.

The log-linearized model can now be reduced just one dynamic equation for the job finding rate (see Appendix A2 for a derivation):

$$\begin{aligned} \mathbb{E}_s \widehat{\eta}_{s+1} &= \Psi \widehat{\eta}_s, \\ \Psi &\equiv \frac{\phi\gamma^{-1}\mu\chi\beta + \phi\gamma^{-1}\frac{\beta\delta_\theta}{1-\alpha} + w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \phi\gamma^{-1}\mu\beta^2\overline{R}\chi + \phi\gamma^{-1}\beta^2\overline{R}\Theta^F} = \frac{\Psi^N}{\Psi^D}. \end{aligned} \tag{35}$$

Both the numerator, $\Psi^N$, and the denominator, $\Psi^D$, are positive, and while the expression for $\Psi$ seems complicated at a first glance, it turns out to deliver very intuitive results, which we present below.[25]

**Determinacy around the intended steady state under inelastic real wages:** How does the presence of incomplete markets impact on the possibility of local self-fulfilling equilibria? Recall the interaction between demand and supply by which higher job uncertainty lowers aggregate demand, which in turn reduces the incentives to post vacancies. The reduction in vacancies in turn reduces the job finding rate, further increasing unemployment risk. Monetary policy must intervene in this feedback mechanism to rule out the possibility that exogenous changes in beliefs, or "sunspot fluctuations," can generate self-fulfilling equilibria.

The model formalizes the condition under which such fluctuations can occur. For simplicity, we start with a version with inelastic real wages ($\chi = 0$). In this case, $\Theta^F > 0$ and the endogenous earning risk is unambiguously countercyclical. Since the job finding rate is not a state variable, the equilibrium is locally determinate if and only if $\Psi > 1$ , i.e. if and only if:

$$\frac{\phi}{\gamma}\left(\beta^2\overline{R}\Theta^F - \frac{\beta\delta_\theta}{1-\alpha}\right) < \frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1 - \beta\left(1-\omega\right)\right).$$

This condition clarifies the importance of the various market frictions and their interaction. The occurrence of local indeterminacy depends on four types of market frictions present in the model, as well as on monetary policy:

(i) *Price rigidity.* If prices are fully flexible ($\phi = 0$) the equilibrium is always determinate

and $\delta_\theta$ one can find a value $\delta_\theta^*$ such that the same solution is obtained under the restriction that $\delta_\pi = \frac{1}{\beta}$.

[25] $\Psi^N$ is easily seen to be positive since it involves only positive or non-negative terms. The denominator is also confirmed to be positive by insertion of the expression for $\Theta^F$.

since the left-hand side collapses to zero and the right-hand side is strictly positive. The stickier are prices ($\phi > 0$), the more likely is the possibility that the equilibrium becomes locally indeterminate.

(ii) *Imperfect competition.* Under perfect competition ($\gamma \to \infty$) the equilibrium is always determinate, since prices will be flexible. Less substitution across goods (lower $\gamma$) impacts on the determinacy condition symmetrically to larger nominal rigidities.

(iii) *Endogenous earnings risk.* Under inelastic real wages, the endogenous risk parameter collapses to $\Theta^F = \omega\eta\left((\vartheta/w)^{-\mu} - 1\right) \geq 0$. Thus, when real wages are inelastic, a larger endogenous risk wedge unambiguously demands more aggressive monetary policy to ensure local determinacy of the intended equilibrium. When $\Theta^F = 0$, the equilibrium is always locally determinate.

(iv) *Labor adjustment cost.* The term $\frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1 - \beta\left(1 - \omega\right)\right)$ denotes the steady-state marginal cost of hiring a worker today rather than tomorrow, so we can think of it as a labor adjustment cost, i.e. a real labor rigidity. Note that this cost is proportional to the steady-state hiring cost $\frac{\kappa}{q}$.

(v) *Monetary policy.* The more aggressively monetary policy responds to tightness, i.e. the higher $\delta_\theta$, the less likely indeterminacy is to occur.

There are two main differences between the incomplete markets model with endogenous countercyclical earnings risk and the standard model with insurance against idiosyncratic risk. The first is simply that the conditions for determinacy are more stringent under incomplete markets. With complete markets, a sufficient conditions for local determinacy is that $\delta_\pi > 1$ as we have assumed. Intuitively, in the standard NK model, when $\delta_\pi > 1$ self-fulfilling inflationary expectations are ruled out in the vicinity of the intended equilibrium because higher expected inflation leads to lower actual inflation, due to the hike in the nominal interest rate. Under incomplete markets and countercyclical endogenous earnings risk, this condition is no longer sufficient because higher expected inflation also stimulates goods demand which, in turn, is inflationary. Thus, monetary policy needs to be even more aggressive to rule out expectational equilibria.

Secondly, the wedges interact in important ways. As long as monetary policy dominates the endogenous risk wedge, $\Theta^F < \beta\delta_\theta$, the sticky-price wedge and the labor market wedge are irrelevant and the intended equilibrium is locally determinate. However, once the endogenous

risk wedge dominates the monetary policy effect, $\Theta^F > \beta\delta_\theta$, the three wedges all matter and there is complementarity between the sticky-price wedge and the endogenous risk wedge. In particular, market incompleteness, nominal rigidities and risk aversion become complements, making local indeterminacy increasingly likely in combination.

An intriguing insight regards the impact of labor market frictions since the higher is the labor adjustment cost, the *less* likely it is for indeterminacy to happen. Thus, less flexible labor markets imply less amplification. The reason for this is that when it is costly for firms to adjust on the labor margin, they are more likely to adjust prices which neutralizes the feedback mechanism.

The above analysis complements a literature which has studied local determinacy in New Keynesian Models with both forward-looking and "rule-of-thumb households", see for example Galí, López-Salido and Vallés (2003) and Bilbiie (2008). A crucial difference with our environment, however, is that in these models there is no idiosyncratic risk and hence no precautionary savings motive. In our model, the precautionary motive, coupled with endogenous risk, is the key source behind the breakdown of the Taylor principle, as demonstrated above.

**Determinacy around the intended steady state under flexible real wages:** The determinacy condition becomes somewhat more involved when we introduce elastic wages ($\chi > 0$):

$$\frac{\phi}{\gamma}\left(\beta^2\overline{R}\Theta^F - \frac{\beta\delta_\theta}{1-\alpha}\right) - w\chi - \frac{\phi}{\gamma}\mu\beta\left(1-\beta\overline{R}\right)\chi < \frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1-\beta\left(1-\omega\right)\right).$$

Wage responses affect determinacy via three channels. First, it does so via the *endogenous risk channel*. Recall that $\Theta^F = \omega\eta\left(\left(\vartheta/w\right)^{-\mu}-1\right) - \chi\mu\omega\left(1-\eta\right)$. Hence, wage flexibility reduces the countercyclicality of the endogenous risk wedge and makes it more likely that the intended equilibrium is locally determinate. Furthermore, if wage flexibility is sufficiently high that $\Theta^F < 0$, local determinacy is guaranteed as longs as $\delta_\theta \geq 0$.

Second, wage flexibility creates a *marginal cost channel*, as it pushes down wage costs during times of low market tightness, pushing up vacancy posting. This channel comes in via the term $-w\chi$. Finally, wage flexibility generates a *discounting channel*, which enters via the term $-\phi\gamma^{-1}\mu\beta\left(1-\beta\overline{R}\right)\chi$. This term arises only under incomplete markets, but does not require job risk to be endogenous. It emerges due to the Euler equation "discount" on future income (consumption), $\beta\overline{R} < 1$. See McKay, Nakamura and Steinsson (2017) for a discussion of this discount in relation to the "forward guidance puzzle".

Finally, note that through all three channels wage flexibility pushes the model towards the

determinacy region of the parameter space. In conclusion, real wage flexibility is stabilizing in the vicinity of the intended steady state.

**Determinacy around the unemployment trap:** We now consider local determinacy around the unemployment trap. To this end, we exploit that the non-negativity constraint on vacancies binds. Hence, we can drop Equation (32) and set $\eta_s$ equal to 0 (or equal to some lower bound if some frictionless hiring is introduced). Thus, the job finding rate is trivially determined. The Euler equation, log-linearized around the unemployment trap, is given by:

$$0 = \delta_\pi \widehat{\Pi}_s - \mathbb{E}_s \widehat{\Pi}_{s+1}.$$

It follows immediately that the equilibrium is unique if and only if $\delta_\pi > 1$, i.e. the interest rate elasticity with respect to inflation exceeds unity. Thus, the unemployment trap is determinate under a standard Taylor rule which responds more than one-for-one to inflation.

# 4 Fluctuations

We now solve for the local dynamics in the vicinity of the intended steady state in response to aggregate shocks. We focus on the impact of productivity shocks and monetary policy shocks, but it is not difficult to derive the implications for other shocks, such as mark-up shocks or non-fundamental "belief shocks."[26]

## 4.1 Productivity shocks

The model with productivity shocks can be written as:

$$\begin{aligned} \mathbb{E}_s \widehat{\eta}_{s+1} &= \Psi \widehat{\eta}_s - \Omega A_s, \\ A_s &= \rho_A A_{s-1} + \sigma_A \varepsilon_s^A, \\ \Omega &= \frac{(\gamma - 1)/\gamma}{\Psi^D}, \end{aligned}$$

where $\Omega > 0$ since we established above that $\Psi^D > 0$ and $\gamma > 1$. See Appendix A2 for a derivation.

[26] We outline the implications for belief shocks in Appendix A2. For an analysis of technology shocks in the standard New Keynesian model, see Galí (1999).

We concentrate on the determinate case ($\Psi > 1$). Apply the method of undetermined coefficients and guess a solution of the form $\widehat{\eta}_s = \Gamma_\eta^A A_s$. Plugging this guess into the above system of equations yields the following solution:

$$\Gamma_\eta^A = \frac{\Omega}{\Psi - \rho_A}. \tag{36}$$

In the determinacy region of the parameter space, the job finding rate responds positively to productivity shocks, i.e. $\Gamma_\eta^A > 0$. To see why, recall that the numerator of Equation (37) is positive and note that for the denominator to be positive as well, it is required that $\Psi > \rho_A$, which is satisfied automatically in the determinacy given that $\Psi > 1$ and $\rho_A < 1$.

Writing out the solution for $\Gamma_\eta^A$ explicitly gives:

$$\Gamma_\eta^A = \frac{\gamma - 1}{\phi\beta\left(\frac{\delta_\theta}{1-\alpha} - \rho_A\beta\overline{R}\Theta^F\right) + \gamma\frac{\kappa}{q}\frac{\alpha(1-\rho\beta(1-\omega))}{1-\alpha} + \left(\gamma w + \phi\mu\beta\left(1 - \rho_A\beta\overline{R}\right)\right)\chi}.$$

Given $\Gamma_\eta^A > 0$, it holds that $\frac{\partial\Gamma_\eta^A}{\partial\Theta^F} \geq 0$, i.e. a higher value of the endogenous risk parameter $\Theta^F$ amplifies the impact of productivity on the job finding rate.

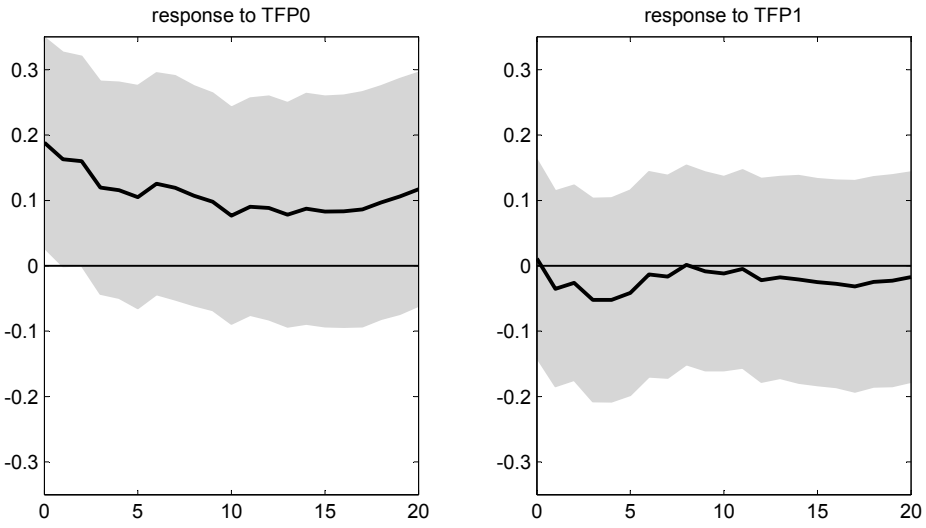
When the endogenous earnings risk is countercyclical, there is complementarity between the endogenous risk wedge and sticky prices, giving rise to amplification. In this case, more aggressive monetary policy dampens the response, since $\frac{\partial\Gamma_\eta^A}{\partial\delta_\theta} \leq 0$, but only when prices are sticky. Elastic real wages dampen the response of the job finding rate to productivity shocks, i.e. $\frac{\partial\Gamma_\eta^A}{\partial\chi} < 0$, since $\beta\overline{R} \leq 1$, $\rho_A \in (-1, 1)$ and $\frac{\partial\Theta^F}{\partial\chi} < 0$.

We can now solve for the inflation rate, guessing a solution of the form $\widehat{\Pi}_s = \Gamma_\Pi^A A_s$. Plugging this guess into the log-linearized Euler equation gives:

$$\Gamma_\Pi^A = \frac{\beta^2\overline{R}\Theta^F\rho_A - \frac{\beta\delta_\theta}{1-\alpha} - \mu\chi\beta\left(1 - \rho_A\beta\overline{R}\right)}{1 - \beta\rho_A}\Gamma_\eta^A.$$

It follows that inflation decreases following a positive technology shock (i.e. $\Gamma_\Pi > 0$) if and only if $\beta^2\overline{R}\Theta^F\rho_A < \frac{\beta\delta_\theta}{1-\alpha} + \mu\chi\beta\left(1 - \rho_A\beta\overline{R}\right)$. In this case, the dominating impact of technology shocks on inflation derives from the decline in marginal costs produced by higher productivity. When this inequality instead is reversed, higher productivity stimulates goods demand sufficiently strongly that inflation actually *rises*. Thus, a strongly countercyclical risk wedge will tend to imply small or even positive inflation responses to productivity shocks, which contrasts with the predictions of the standard NK model (unless productivity shocks are sufficiently non-stationary).

Figure 4: Response of CPI inflation to a positive TFP shock.




Notes: IRF of 400*log(cpit/cpit-1) to change in log TFP as estimated by Fernald http://www.frbsf.org/economic-research/publications/working-papers/2016/wp2016-07.pdf using local projection. The sample starts in 1980 and we included 4 lags. TFP0 (TFP1) refers to Fernald estimate for Total Factor Productivity without (with) control for factor utilization. Shaded areas denote error bands of two standard deviations.

The possibility that increased productivity is inflationary is not a mere theoretical curiosity. In Figure 4, we show the impulse response of CPI inflation to TFP shocks where the latter correspond to those estimated by Fernald and Wang (2016). Using local projection, we regressed (400 times) quarterly (log) changes in the CPI on TFP (log) growth (times 100) for a sample that starts in 1980. Depending on whether one controls for movements in factor utilization or not, higher TFP either leaves inflation unchanged or gives rise to higher inflation. While the empirical results come with a fair amount of uncertainty, they do suggest that a positive inflation response is not simply an odd feature of our model.[27]

## 4.2 Monetary policy shocks

Above we have shown that the systematic components of monetary policy are crucial for the impact of technology shocks. We now consider the effects of monetary policy shocks. In particular,

[27] The result holds also for the core PCE and here the positive response holds regardless of the TFP measure. The results also hold true for a sample period that starts in 1984, the sample split that Fernald and Wang (2016) focus upon.

we introduce an exogenous shock $e_t^R$ to the interest rate rule:

$$R_s = \max\left\{\overline{R}\left(\frac{\Pi_s}{\overline{\Pi}}\right)^{\delta_\pi}\left(\frac{\theta_s}{\overline{\theta}}\right)^{\delta_\theta}\exp\left(e_t^R\right),1\right\}.$$

We assume that $e_t^R$ follows an AR(1) process with persistence parameter $\rho_R$. The solutions for the job finding rate and the inflation rate when log-linearizing the model around the intended steady state are:

$$\begin{aligned}
\widehat{\eta}_s &= \Gamma_\eta^R e_s^R, \\
\widehat{\Pi}_s &= \Gamma_\Pi^R e_s^R, \\
\Gamma_\eta^R &= \frac{-\phi}{\phi\beta\left(\frac{\delta_\theta}{1-\alpha}-\rho_R\overline{R}\Theta^F\right)+\frac{\gamma}{\beta}\frac{\kappa}{q}\frac{\alpha(1-\beta\rho_R(1-\omega))}{1-\alpha}+\left(\frac{\gamma}{\beta}w+\phi\mu\left(1-\rho_R\beta\overline{R}\right)\right)\chi}, \\
\Gamma_\Pi^R &= \frac{\beta\left(\beta\overline{R}\Theta^F\rho_R-\frac{\delta_\theta}{1-\alpha}-\chi\left(1-\rho_R\beta\overline{R}\right)\right)\Gamma_\eta^R-\beta}{1-\beta\rho_R}.
\end{aligned}$$

Following the same logic as above, we can verify that $\Gamma_\eta^R < 0$. That is, a contractionary monetary policy shock triggers a decline in the job finding rate and hence in output, as in the standard NK model. This decline is amplified by the presence of countercyclical risk, since $\frac{\partial\Gamma_\eta^R}{\partial\Theta^F} > 0$ provided that there is some persistence in the monetary policy shocks ($\rho_R > 0$). Intuitively, the boom in demand created by a monetary expansion reduces idiosyncratic risk, creating a further boom in demand.

Hence, market incompleteness will tend to provide a more powerful role for monetary policy in the case where the endogenous earnings risk is countercyclical. This increased importance of monetary policy derives both through the systematic parts of policy as well as through the impact of monetary policy shocks.

## 4.3 Discussion

We have shown above how countercyclical endogenous earnings risk provides an amplification mechanism in a model combining HANK and SAM. Key to this amplification is the precautionary savings motive and the interaction between the supply and the demand side of the economy generated by the endogenous earnings risk and nominal rigidities.

A possible reservation regarding these results is that the introduction of capital accumulation could potentially neutralize the amplification mechanism. In particular, when the model implies

amplification, one might think that the downward pressure on real interest rates stimulates investment in real capital which, in turn, neutralizes the amplification. However, this might not necessarily be the case because depressed goods demand (in bad times) also means lower return on capital investment which magnifies the amplification.

It is not possible to solve the model in closed form when we introduce capital. In Appendix A7, however, we provide some numerical results for a calibrated version of the model extended with capital accumulation. We assume that capital is owned by the firms and that the production function is Cobb-Douglas. Entrepreneurs are assumed to be risk averse with preferences given in (1). For simplicity we set the real wage constant, an outcome that corresponds to the Nash bargaining solution when the value of leisure is sufficiently high and firms' bargaining power is close to one.

Figure 9 in Appendix A7 illustrates the impact of a one percent negative productivity shock on unemployment in the economy with and without capital accumulation, comparing the outcome of the model with sticky prices and countercyclical endogenous risk with versions of the model that assume either flexible prices or exogenous risk. We find that the introduction of capital accumulation preserves the amplification mechanism, or even makes it slightly stronger. In other words, it is the demand effect that dominates.

# 5 Implications for the Zero Lower Bound

Our analysis thus far has focused on the implications of the endogenous risk channel when the economy is away from the ZLB on the nominal interest rate. In this section, we analyze how the channel impacts on paths into the ZLB, and economic outcomes once the ZLB is reached.

## 5.1 Contractionary Shocks and the ZLB

A recent literature has emerged on the effects of the Zero Lower Bound (ZLB) in the New Keynesian model, see e.g. Christiano, Eichenbaum and Rebelo (2011), Krugman and Eggertsson (2012) and Farhi and Werning (2013). Often, such analyses start off from a premise that some exogenous and transitory shock brings the economy temporarily to the ZLB. The specific shock introduced for this purpose is typically an exogenous shock to the discount factor, making agents temporarily more patient. The increase in patience drives down aggregate demand, putting downward pressure on inflation and the real interest rate. Via the interest rate rule, this results in a decline in the nominal interest rate, which may hit the ZLB if the shock is large enough

(and at that point induces a potentially significant recession in the economy).

To appreciate the purpose of this specific shock, it helps to note that more conventional recessionary shocks, such as negative productivity shocks, typically will not lead to a decline in the nominal interest rate. There are two reasons for this. First, recessionary shocks reduce aggregate income and in a representative-agent model, lower current income (relative to expected future income) reduces households' desire to save inducing upward pressure on real and nominal interest rates, see e.g. Galí (2015, Chapter 3). A negative technology shock additionally increases real marginal costs, which puts further upward pressure on inflation and, via the Taylor rule, also the nominal interest rate. Thus, in a standard NK model without other sources of shocks, *expansionary* rather than *recessionary* technology shocks tend to be required to produce a decline in the nominal interest rate. For that reason, much research in the NK literature has introduced discount factor shocks when studying ZLB dynamics.

The precautionary savings mechanism that arises under endogenous risk can radically alter the cyclicality of the real interest rate, avoiding the need for discount factor shocks. Mechanically, the endogenous risk wedge acts as a shock to the discount factor in the Euler equation, but is determined endogenously rather than exogenously. Assume that $\Theta^F > 0$. As economic conditions worsen, the risk of becoming unemployed increases, driving down aggregate demand and increasing agents' desire to save. If the precautionary savings mechanism is strong enough, the nominal interest rate may decline, as argued by for example Werning (2015).[28]

Here, we can exploit the solution to the full model to obtain an explicit condition for the nominal interest rate to decline in response to a negative productivity shock. For simplicity, let us assume that monetary policy only responds to inflation ($\delta_\theta = 0$) and abstract from monetary policy shocks.[29] The log-linearized interest rate rule is given as

$$\widehat{R}_s = \delta_\pi \widehat{\Pi}_s = \delta_\pi \Gamma^A_\Pi A_s,$$

where $\delta_\pi > 1$. In the previous subsection, we have shown that $\Gamma^A_\Pi$ is negative when $\Theta^F = 0$. That is, under complete markets (or exogenous earning risk) the inflation rate, and hence the nominal interest rate, responds positively to a negative technology shock. However, when $\Theta^F >$

[28] The working paper version of Ravn and Sterk (2017) make a similar point based on numerical simulations and, but do not consider productivity shocks. Werning (2015) presents analytical arguments, but not a fully fledged model.

[29] We further assume that $\Gamma_\eta > 0$, i.e. the job finding rate responds positively to a positive productivity shock. As shown above, this is always the case in the determinacy region of the parameter space, and may be the case in the indeterminacy region.

$\frac{\mu\chi}{\beta\overline{R}\rho}\left(1-\rho\beta\overline{R}\right)$, i.e. when markets are sufficiently incomplete and the endogenous earnings risk is countercyclical, $\Gamma_{\Pi}^{A}$ is positive. Under this condition, a negative technology shock drives down inflation and the nominal interest rate. If the shock is large enough, the ZLB may become binding.

## 5.2 Understanding Missing Deflation

Although inflation has been moderate in the aftermath of the financial crisis, no country has experienced persistent deflation. This is not easily reconciled with the standard NK model: Under the assumption of complete markets ($\mathbf{\Theta}^{SS}(\eta)=0$), the deterministic steady-state real interest rate is given by $R/\Pi=1/\beta$ and it follows that, when the ZLB binds in a steady state, the gross inflation rate must equal $\beta<1$. Temporary episodes at the ZLB will be even more deflationary than this since the stochastic Euler equation in that case will only be satisfied as long as $\Pi<\beta$ during the ZLB regime.[30] It is important to notice that these implications are independent of the arguments that enter the interest rate rule.

The incomplete markets NK model has different implications. As explained earlier, the steady-state real interest rate under incomplete markets is:

$$\frac{R}{\Pi}=\frac{1}{\beta\mathbf{\Theta}^{SS}(\eta)}<\frac{1}{\beta},$$

which implies that the steady-state real interest rate depends on labor market conditions. When the ZLB binds, the steady-state Euler equation and the policy rule for the interest rate imply that the following two conditions must be satisfied in a liquidity trap ($LT$):

$$\begin{aligned}\Pi^{LT} &= \beta\mathbf{\Theta}^{SS}\left(\eta^{LT}\right)>\beta,\\ \Pi^{LT} &< \overline{\Pi}\theta^{\delta_\theta/\delta_\pi}\overline{R}^{-1/\delta_\pi}\left(\eta^{LT}\right)^{-(\delta_\theta/\delta_\pi)/(1-\alpha)}.\end{aligned}$$

Notice that if $\delta_\theta=0$, the policy rule implies that $\Pi^{LT}<\overline{\Pi R}^{-1/\delta_\pi}<1$, so that the liquidity trap is deflationary, given that in the intended steady state $\Pi=\overline{\Pi}=1$ and $R=\overline{R}>1$. When $\delta_\theta>0$, however, inflation may be positive or negative in the liquidity trap. In particular, steady-state inflation is likely to be positive if $\left(\vartheta/w\left(\eta^{LT}\right)\right)^{-\mu}\gg 1$ and wages are not too responsive to

[30] Suppose that the ZLB regime persists with probability $p$ while the intended steady-state is absorbing. In that case, the inflation rate during the ZLB episode is determined as $\Pi^{LT}=\beta\left(p+(1-p)\left(c^{I}/c^{LT}\right)^{-\mu}\right)$ where $\Pi^{LT}$ is the inflation rate during the liquidity trap, $c^{I}$ is consumption in the intended steady-state and $c^{LT}$ is consumption in the liquidity trap. This condition implies $\Pi^{LT}<\beta$ as long as $c^{I}>c^{LT}$.

the job finding rate, i.e. when the endogenous risk wedge is sufficiently countercyclical. Intuitively, under these circumstances, deteriorating labor market conditions (worsening tightness) induces both lower nominal interest rates and lower goods demand which in turn implies a further decline in tightness and in nominal rates the end-product of which may be that the ZLB may be reached at a positive inflation rate.

## 5.3 Paradoxes at the Zero Lower Bound

It is well known that at the ZLB, the representative-agent NK model has some paradoxical properties, see e.g. Eggertsson (2010), Eggertsson and Krugman (2012) and Werning (2012). One prominent example is the "supply shock paradox": at the ZLB, positive shocks to the supply side of the economy can trigger a contraction in real activity.[31]

The paradox arises from the fact that a positive supply shock pushes down production costs and hence inflation. The increase in inflation, in turn, creates paradoxical effects which can be understood from the consumption Euler equation. Consider, for simplicity, the complete-markets Euler equation under perfect foresight at the ZLB:

$$\left(\frac{c_{s+1}}{c_s}\right)^{\mu} = \beta \frac{1}{\Pi_{s+1}}.$$

The effect of a decline in expected inflation, at the ZLB, is that the real interest rate, $\frac{1}{\Pi_{s+1}}$, increases. The above Euler equation makes clear that this implies an increase in expected consumption growth, $c_{s+1}/c_s$. Given that the decline in inflation is transitory however, an increase in expected consumption growth implies a *decline* in the current level of consumption, i.e. an economic contraction.[32]

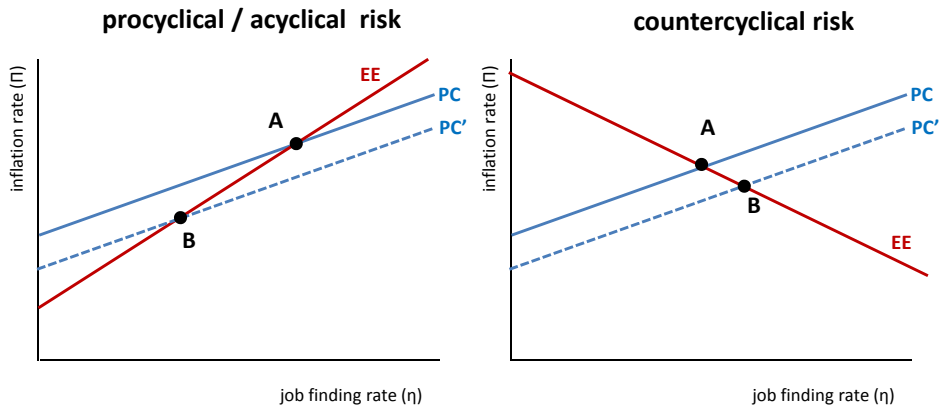
The joint presence of incomplete markets and countercyclical earnings risk, however, can overturn these results. Mechanically, the endogenous risk wedge in the Euler equation can absorb the effect of a decline in the real interest rate. Intuitively, an increase in output implies an increase in hiring, which reduces the precautionary savings motive. This makes an expansion in output compatible with an increase in the real interest rate.

We now formalize these arguments. Suppose that the economy fluctuates discretely between

[31] Another important and closely related example is the "paradox of flexibility" which states that, at the ZLB, a higher degree of price flexibility creates a larger drop in output.

[32] Throughout this subsection, we consider equilibria which ultimately lead to the intended steady state. Properties of equilibria leading to the liquidity trap steady state can be very different, see e.g. Mertens and Ravn (2014).

Figure 5: Supply Shock Paradox at the ZLB: illustration.




a "depressed state" at which the ZLB binds, and a "normal state" which coincides with the intended steady state. Let $p \in (0,1)$ be the probability that the ZLB regime persists and let the normal state be absorbing. In Appendix A4 we derive the relation between inflation and the job finding rate implied by the Euler equation, illustrated by lines labeled "**EE**" in Figure 4 . The slope is given by:

$$\frac{d\widehat{\Pi}_s}{d\widehat{\eta}_s} = \frac{\mu\chi}{p}\left(1-\beta\overline{R}p\right) - \beta\overline{R}\Theta^F.$$

Under acyclical risk ($\Theta^F = 0$), or under procyclical endogenous earnings risk $\left(\Theta^F < 0\right)$ the elasticity is positive since $\mu\chi > 0$ and $\beta\overline{R}p < 1$. Thus, any additional shock which reduces inflation must create a labor market contraction. As explained above, this is the source of the paradox. However, when $\Theta^F > \frac{\mu\chi}{p}(\beta^{-1}\overline{R}^{-1} - 1)$, i.e. when the endogenous earnings risk is highly countercyclical, the slope is negative. In that case, a reduction in inflation coincides with a labor market expansion.

In order to study explicitly the effect of a change in productivity, consider now the supply side of the economy. The Phillips Curve implies a positive relation between inflation and the job finding rate, see Appendix A4 for details. The lines in Figure 4 labeled "**PC**" illustrate this relation. An increase in productivity shifts down the **PC** curve and moves the equilibrium from point A to point B.

The left panel of Figure 4 depicts an economy with acyclical/procyclical risk and illustrates the paradox that arises also under complete markets: the increase in productivity reduces the job finding rate, and hence employment. The right panel illustrates a case with a downward-sloping **EE** curve, due to countercyclical risk. In this case, the job finding rate increases in response to the productivity increase. Thus, the presence of incomplete markets and countercyclical risk

can overturn the supply shock paradox.

# 6 An Empirical Perspective

A central implication of the model is that the presence of endogenous earnings risk can create either dampening or amplification of fluctuations. Dampening arises under procyclical risk ($\Theta^F < 0$), whereas amplification occurs under countercyclical risk ($\Theta^F > 0$). We argued earlier that countercylical endogenous risk is likely to be the empirically relevant case but now we want to show this more formally.

To do so we exploit the analytical formula for the endogenous risk parameter $\Theta^F$ in Equation (31). Consider a time period of one month and evaluate the steady-state job finding rate $\eta$ and the job loss rate $\omega$, as the average of their counterparts in the Current Population Survey (CPS) over the period January 1990 until September 2017.[33] Recall further that $1 - \vartheta/w = 1 - c_u/c_e$ is the decline in consumption upon job loss. Following Karabarbounis and Chodorow-Reich (2017), we assume that consumption drops 20 percent upon job loss. However, we also consider a much smaller drop of only 5 percent. For the risk aversion parameter $\mu$ we consider both $\mu = 0.5$ and $\mu = 2$, as estimates of this parameters vary across studies in the literature.

The final parameter that matters is the wage flexibility parameter, $\chi = \frac{\partial \ln w_s}{\partial u_s} \frac{\partial u_s}{\partial \ln \eta_s}$. We can obtain the second term by differentiating the transition equation for unemployment with respect to the (log of the) job finding rate. Evaluated at the steady state this gives $\frac{\partial u_s}{\partial \ln \eta_s} = -\eta u - \omega\eta\,(1-u)$, which we evaluate using CPS data. The semi-elasticity of the wage with respect to unemployment, $\frac{\partial \ln w_s}{\partial u_s}$, has been estimated in several studies. Gertler, Huckfeldt and Trigari (2016) estimate this elasticity to be $\frac{\partial \ln w_s}{\partial u_s} = -0.16$ for job stayers (see their Table 2, fourth column). We take this number as our baseline, since $\Theta^F$ captures the expected wage of those currently employed, in the event they remain employed. However, we also consider a much larger elasticity of $-1.5$, which is in the ballpark of the estimates which Gertler et al. estimate specifically for new hires from unemployment ($-0.164$) and job switchers ($-2.085$).[34]

Table 1 shows the results. The countercyclical effect of unemployment risk clearly dominates the procyclical effect of wage risk (i.e. $\Theta^F > 0$). Only when we assume *both* a small consumption

[33] In particular, we measure $\eta_t$ as the unemployment-to-employment transition rate, $\omega_t = \frac{u_t - u_{t-1}(1-\eta_t)}{(1-u_{t-1})(1-\eta_t)}$, given a series for the unemployment rate $u_t$, consistent with the timing assumptions in our model.

[34] We also estimated $\chi$ directly by running a regression of $w_s$ on the job finding rate $\eta_s$, and a time trend. Here, we measured $w_s$ as average hourly earnings of production and nonsupervisory employees, deflated by the CPI. While results varied across specifications, the largest wage elasticity we found was $\widehat{\chi} = 0.03$ which corresponds to about $\frac{\partial \ln w_s}{\partial u_s} = -1.5$.

Table 1: Cyclicality of earnings risk in the log-linearized model.

I. baseline wage elasticity ($\frac{\partial \ln w_t}{\partial u_t} = -0.16$)

| consumption loss upon job loss | baseline (20%) | | low (5%) | |
|---|---|---|---|---|
| coefficient of risk aversion | $\mu = 0.5$ | $\mu = 2$ | $\mu = 0.5$ | $\mu = 2$ |
| 1) $\Theta^F$: unemployment | 0.0436 | 0.2079 | 0.0096 | 0.0399 |
| 2) $\Theta^F$: wage | $-0.0017$ | $-0.0067$ | $-0.0017$ | $-0.0067$ |
| 3) $\Theta^F$: total | 0.0420 | 0.2013 | 0.0079 | 0.0333 |

II. high wage elasticity ($\frac{\partial \ln w_t}{\partial u_t} = -1.5$)

| consumption loss upon job loss | baseline (20%) | | low (5%) | |
|---|---|---|---|---|
| coefficient of risk aversion | $\mu = 5$ | $\mu = 2$ | $\mu = 0.5$ | $\mu = 2$ |
| 1) $\Theta^F$: unemployment | 0.0436 | 0.2079 | 0.0096 | 0.0399 |
| 2) $\Theta^F$: wage | $-0.0167$ | $-0.0666$ | $-0.0167$ | $-0.0666$ |
| 3) $\Theta^F$: total | 0.0270 | 0.1413 | $-0.0071$ | $-0.0267$ |

Notes: $\Theta^F > 0$ implies countercyclical earnings risk. 1): $\omega\eta\left((\vartheta/w)^{-\mu} - 1\right)$, 2): -$\chi\mu\omega\left(1-\eta\right)$, 3): $\omega\eta\left((\vartheta/w)^{-\mu} - 1\right) - \chi\mu\omega\left(1-\eta\right)$. All results have been multiplied by 100.

drop (5 percent) and a very elastic wage ($\frac{\partial \ln w_s}{\partial u_s} = -1.5$), do we find that the effect of wage risk slightly dominates. Given that these values are relatively unlikely in the light of most studies in the literature, we conclude that countercyclical earnings risk is the more relevant case.

To get a sense of the magnitudes, note that an increase in the wedge in Equation (31) has the same partial-equilibrium effect on consumption growth of the employed as a change in the real interest rate. Consider also the fact that during the Great Recession, the job finding rate fell by about fifty percent. Given this decline, the baseline parametrization under $\mu = 2$ implies a change in the endogenous risk wedge which is equivalent to an increase in the annual real interest rate of about 120 basis points.

An alternative way of evaluating the key mechanism of the models is to consider the implied relation between the real interest rate and market tightness, as plotted in Figure 1. Consider the model with only productivity shocks. Using the log-linearized Euler equation, the real interest rate can be expressed as:

$$\widehat{R}_s^r = (1-\alpha)\left(\Theta^F \beta \overline{R} \rho_A - \left(1 - \beta \overline{R} \rho_A\right)\mu\chi\right)\widehat{\theta}_s$$

where $\widehat{R}_s^r \equiv \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1}$. This equation provides a direct relation between market tightness and the real interest rate, which we can confront with the data. Recall that $\widehat{R}_s^r$ and $\widehat{\theta}_s$ are positively correlated in the data. The above expression implies that in the model this can only be the case

if $\Theta^F$ positive and sufficiently large. Again, the data suggest that countercyclical earnings risk is the more relevant case.

# 7 Conclusion

We have proposed a simple and intuitive heterogeneous-agents New Keynesian (NK) model with endogenous unemployment, and highlighted that the interaction between market frictions can give rise to belief-driven equilibria. Moreover, the interaction between these frictions produces potentially a significant amount of amplification of shocks to the economy. This amplification occurs when the endogenous earnings risk is countercyclical and we have argued that this is the empirically plausible case. The essence of the interaction is that incomplete markets produce movements in aggregate demand in response to fluctuations in the job finding rate which impact on the supply side when there are nominal rigidities and creates a feedback mechanism. In particular, weak labor demand produces low goods demand which in itself produces low labor demand. The combination of HANK and SAM therefore has fundamental consequences and puts labor markets in the centre of the amplification and transmission mechanism.

We have also shown that the new NK model with countercyclical earnings risk can resolve a number of puzzles that have arisen in the macroeconomic literature. These involve the existence of persistent low growth equilibria with low but positive inflation, the impact of supply shocks on inflation dynamics, and various paradoxes at the ZLB. Intriguingly, the model can also provide a coherent framework for understanding the positive relationship between real interest rates and labor market tightness which can be observed in the US.

In Appendix A9, we demonstrate that under incomplete markets the NK model becomes useful to analyze the link between monetary policy and financial asset prices. While we limit the analysis to simple analytical exercises, it would be interesting to evaluate the extent to which a full-scale heterogeneous-agents NK can explain observed asset prices. Vice versa, financial markets data may be useful to impose empirical discipline on the new generation of NK models.

Throughout the analysis, we have assumed that government policies are summarized by a simple interest rate rule, subject to the zero lower bound. It would be interesting to use the framework to obtain insights into the stabilization effects of other government policies, such as fiscal policy or labor market policies. Also, the framework could be used to consider optimal policies. We leave these issues for future research.

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# Appendix

## A1. Steady-state Nash bargaining solution

The steady-state expressions of the asset-poor households' surplus and value functions are:

$$\begin{aligned} V^e\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)\right) &= \frac{w^{1-\mu}}{1-\mu}-\zeta+\beta\omega\left(1-\eta\right)V^u, \\ V^u\left(1-\beta\left(1-\eta\right)\right) &= \frac{\vartheta^{1-\mu}}{1-\mu}+\beta\eta V^e, \end{aligned}$$

where we have exploited that in equilibrium the asset-poor households are the same and consume their incomes. Now substitute out $V^u$ in the first equation:

$$V^e\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)\right)=\frac{w^{1-\mu}}{1-\mu}-\zeta+\frac{\beta\omega\left(1-\eta\right)}{1-\beta\left(1-\eta\right)}\left(\frac{\vartheta^{1-\mu}}{1-\mu}+\beta\eta V^e\right).$$

$$V^e\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)-\frac{\beta\omega\left(1-\eta\right)}{1-\beta\left(1-\eta\right)}\beta\eta\right)=\frac{w^{1-\mu}}{1-\mu}-\zeta+\frac{\beta\omega\left(1-\eta\right)}{1-\beta\left(1-\eta\right)}\frac{\vartheta^{1-\mu}}{1-\mu},$$

We can now express the two values as functions of $\eta$ and $w$:

$$\begin{aligned} V^e\left(\eta,w\right) &= \frac{\frac{w^{1-\mu}}{1-\mu}-\zeta+\frac{\beta\omega(1-\eta)}{1-\beta(1-\eta)}\frac{\vartheta^{1-\mu}}{1-\mu}}{1-\beta\left(1-\omega\left(1-\eta\right)\right)-\frac{\beta\omega(1-\eta)\beta\eta}{1-\beta(1-\eta)}} \\ V^u\left(\eta,w\right) &= \frac{\frac{\vartheta^{1-\mu}}{1-\mu}+\beta\eta V^e\left(\eta,w\right)}{1-\beta\left(1-\eta\right)} \end{aligned}$$

The first-order condition to the Nash Bargaining problem is given by

$$\left(1-\upsilon\right)S^e=\upsilon S^f\ ,$$

or,

$$\left(1-\upsilon\right)\left(V^e\left(\eta,w\right)-V^u\left(\eta,w\right)\right)=\upsilon\kappa\eta^{\alpha/(1-\alpha)}\ .$$

$$\left(V^e\left(\eta,w\right)-V^u\left(\eta,w\right)\right)=\frac{\upsilon}{1-\upsilon}\kappa$$

The above is an equation in two variables, which implicitly defines the wage as a function of the job finding rate, i.e the function $w(\eta)$.

**Basic properties**: Consider the special case in which $\eta=0$. From the Nash bargaining solution it follows that the wage must satisfy $V^e\left(0,w(0)\right)=V^u\left(0,w(0)\right)=\frac{\frac{\vartheta^{1-\mu}}{1-\mu}}{1-\beta}$. It follows that

$\frac{w(0)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta$ and hence $w(0) > \vartheta$ whenever $\zeta > 0$.

At the other extreme, under $\eta = 1$ we get from the Nash Bargaining solution $V^e(1,w) = V^u(1,w) + \frac{\upsilon\kappa}{1-\upsilon}$. Also, the worker value functions imply that $V^e(1,w) - V^u(1,w) = \frac{w(1)^{1-\mu}}{1-\mu} - \zeta - \frac{\vartheta^{1-\mu}}{1-\mu}$. It follows that $\frac{w(1)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta + \frac{\upsilon}{1-\upsilon}\kappa$ and hence $w(1) > w(0)$ , $V^e(1,w(1)) > V^e(0,w(0))$ and $V^u(1,w) > V^u(0,w)$.

Finally, consider a case in which the worker has no bargaining power ($\upsilon = 0$). It follows from the Nash bargaining solution that in this case $V^e(\eta,w) = V^u(\eta,w)$ which implies that $\frac{w(\eta)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta$. As a result, the real wage does not depend of $\eta$, i.e. the real wage is sticky.

## A2. Log-linearized model

### Nash Bargaining block

The first-order condition to the Nash bargaining problem, together with the asset-poor workers' value functions are given by:

$$\begin{aligned}
(1-\upsilon)\left(V_s^e - V_s^u\right) &= \upsilon\kappa\eta_s^{\alpha/(1-\alpha)}, \\
V_s^e &= \frac{w_s^{1-\mu}}{1-\mu} - \zeta + \beta\mathbb{E}_s\omega\left(1-\eta_{s+1}\right)V_{s+1}^u + \beta\mathbb{E}_s\left(1-\omega\left(1-\eta_{s+1}\right)\right)V_{s+1}^e, \\
V_s^u &= \frac{\vartheta^{1-\mu}}{1-\mu} + \beta\mathbb{E}_s\left(1-\eta_{s+1}\right)V_{s+1}^u + \beta\mathbb{E}_s\eta_{s+1}V_{s+1}^e.
\end{aligned}$$

After log-linearization, the above system can be written in the following form:

$$\boldsymbol{A}\begin{bmatrix} \widehat{V}_s^e \\ \widehat{V}_s^u \\ \widehat{w}_s \end{bmatrix} + \boldsymbol{B}\widehat{\eta}_s = \mathbb{E}_s\boldsymbol{C}\begin{bmatrix} \widehat{V}_{s+1}^e \\ \widehat{V}_{s+1}^u \\ \widehat{w}_{s+1} \end{bmatrix} + \mathbb{E}_s\boldsymbol{D}\widehat{\eta}_{s+1}$$

where $\boldsymbol{A}$ and $\boldsymbol{C}$ are $3\times 3$ matrices and $\boldsymbol{B}$ and $\boldsymbol{D}$ are $3\times 1$ vectors, all consisting of parameter values. Note that none of the variables $\widehat{V}_s^e$, $\widehat{V}_s^u$ and $\widehat{w}_s$ is a state variable. Provided that $\widehat{\eta}_s$ follows some linear law of motion and given the law of motion for $A_s$, we can apply the method of undetermined coefficients to find solutions for $\widehat{V}_s^e$, $\widehat{V}_s^u$ and $\widehat{w}_s$ as linear functions of $\widehat{\eta}_s$. We denote the solution for the wage as $\widehat{w}_s = \chi\widehat{\eta}_s$, where it follows that $\chi$ is a function of the parameters that enter $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$.

**Monetary Policy rule, Euler equation, Phillips Curve**

The log-linerarized monetary policy rule is given by:

$$\widehat{R}_s = \delta_\pi \widehat{\Pi}_s + \delta_\theta \widehat{\theta}_s.$$

Next, consider the Euler equation of the employed households. Exploiting the fact that in Equilibrium $c_{e,s} = w_s$ and $c_{u,s} = \vartheta$, we can express the employed workers' Euler equation, Equation (21), as:

$$w_s^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left( \omega \left( 1 - \eta_{s+1} \right) \vartheta^{-\mu} + \left( 1 - \omega \left( 1 - \eta_{s+1} \right) \right) w_{s+1}^{-\mu} \right),$$

and note that in the intended steady state we obtain $w^{-\mu} = \beta \overline{R} \left( \omega \left( 1 - \eta \right) \vartheta^{-\mu} + \left( 1 - \omega \left( 1 - \eta \right) \right) w^{-\mu} \right)$. Log-linearizing the above equation around the intended steady state gives:

$$\begin{aligned}
-\mu \widehat{w}_s &= \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \overline{R} \omega \eta \left( \vartheta / w \right)^{-\mu} \mathbb{E}_s \widehat{\eta}_{s+1} + \beta \overline{R} \omega \eta \mathbb{E}_s \widehat{\eta}_{s+1} - \mu \beta \overline{R} \left( 1 - \omega \left( 1 - \eta \right) \right) \mathbb{E}_s \widehat{w}_{s+1}, \\
&= -\mu \beta \overline{R} \mathbb{E}_s \widehat{w}_{s+1} + \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \overline{R} \omega \eta \left( \left( \vartheta / w \right)^{-\mu} - 1 \right) \mathbb{E}_s \widehat{\eta}_{s+1} + \mu \beta \overline{R} \omega \left( 1 - \eta \right) \mathbb{E}_s \widehat{w}_{s+1}, \\
&= -\mu \beta \overline{R} \mathbb{E}_s \widehat{w}_{s+1} + \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \overline{R} \Theta^F \mathbb{E}_s \widehat{\eta}_{s+1},
\end{aligned}$$

where $\Theta^F = \omega \eta \left( \left( \vartheta / w \right)^{-\mu} - 1 \right) - \chi \mu \omega \left( 1 - \eta \right)$ and where we used that $\widehat{w}_s = \chi \widehat{\eta}_s$.

Next, consider the firms' price setting condition, which can be written as:

$$\begin{aligned}
& \phi \left( \Pi_s - 1 \right) \Pi_s - \phi \mathbb{E}_s \Lambda_{s,s+1} \frac{y_{s+1}}{y_s} \left( \Pi_{s+1} - 1 \right) \Pi_{s+1} \\
= \; & 1 - \gamma + \frac{\gamma}{\exp\left( A_s \right)} \left( w_s + \kappa \eta_s^{\alpha/(1-\alpha)} - \left( 1 - \omega \right) \kappa \mathbb{E}_s \Lambda_{s,s+1} \eta_{s+1}^{\alpha/(1-\alpha)} + \lambda_{v,s} \right).
\end{aligned}$$

and note that at the intended steady state $\lambda_{v,s} = 0$ and $\Lambda_{s,s+1} = \beta$. Log-linearizing the equation around the intended steady state with $\Pi = 1$ gives:

$$\frac{\phi}{\gamma} \widehat{\Pi}_s - \frac{\phi}{\gamma} \beta \mathbb{E}_s \widehat{\Pi}_{s+1} = w \chi \widehat{\eta}_s + \frac{1 - \gamma}{\gamma} A_s + \frac{\kappa}{q} \left( \frac{\alpha}{1 - \alpha} \widehat{\eta}_s - \frac{\alpha \beta \left( 1 - \omega \right)}{1 - \alpha} \mathbb{E}_s \widehat{\eta}_{s+1} - \beta \left( 1 - \omega \right) \mathbb{E}_s \widehat{\Lambda}_{s,s+1} \right),$$

where we have substituted out the wage using $\widehat{w}_s = \chi \widehat{\eta}_s$.

**Reducing the model**

Under the the two assumptions ($\delta_\pi = \frac{1}{\beta}$ and risk-neutrality of the equity investors) and in the absence of productivity shocks, the log-linearized Euler equation and pricing condition become:

$$\begin{aligned} -\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} &= \widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1} + \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s - \beta^2\overline{R}\Theta^F\mathbb{E}_s\widehat{\eta}_{s+1} \\ w\chi\widehat{\eta}_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) &= \frac{\phi}{\gamma}\left(\widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1}\right) \end{aligned}$$

where in the first equation we have substituted out the interest rate using $\widehat{R}_s = \delta_\pi\widehat{\Pi}_s + \delta_\theta\widehat{\theta}_s$, and tightness using $\widehat{\theta}_s = \frac{\widehat{\eta}_s}{1-\alpha}$. Using the first equation to substitute out $\widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1}$ in the second equation gives:

$$w\chi\widehat{\eta}_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) = \frac{\phi}{\gamma}\left(-\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} - \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s + \beta^2\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1}\right).$$

Collecting terms gives:

$$\mathbb{E}_s\widehat{\eta}_{s+1} = \Psi\widehat{\eta}_s,$$

where

$$\Psi = \frac{\frac{\phi}{\gamma}\mu\chi\beta + \frac{\phi}{\gamma}\frac{\beta\delta_\theta}{1-\alpha} + w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\overline{R}\chi + \frac{\phi}{\gamma}\beta^2\overline{R}\Theta^F}.$$

**Productivity shocks**

With productivity shocks the model becomes:

$$\begin{aligned} w\chi\widehat{\eta}_s + \frac{1-\gamma}{\gamma}A_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) & \\ = \frac{\phi}{\gamma}\left(-\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} - \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s + \beta^2\overline{R}\Theta^F\mathbb{E}_s\widehat{\eta}_{s+1}\right), & \\ A_s = \rho_A A_{s-1} + \sigma_A\varepsilon_s^A, & \end{aligned}$$

which we can rewrite as

$$\left(\frac{\kappa}{q}\frac{\alpha\beta\left(1-\omega\right)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\overline{R}\chi + \frac{\phi}{\gamma}\beta^2\overline{R}\Theta^F\right)\mathbb{E}_s\widehat{\eta}_{s+1} = \left(w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha} + \frac{\phi}{\gamma}\mu\chi\beta + \frac{\phi}{\gamma}\frac{\beta\delta_\theta}{1-\alpha}\right)\widehat{\eta}_s - \frac{\gamma-1}{\gamma}A_s$$

which gives

$$\begin{aligned}\mathbb{E}_s\widehat{\eta}_{s+1} &= \Psi\widehat{\eta}_s - \Omega A_s,\\ A_s &= \rho_A A_{s-1} + \sigma_A \varepsilon_s^A,\end{aligned}$$

where

$$\Omega = \frac{(\gamma-1)/\gamma}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\overline{R}\chi + \frac{\phi}{\gamma}\beta^2\overline{R}\Theta^F}.$$

**Monetary policy shocks**

Now consider the model with monetary policy shocks. The log-linearized model, assuming again risk-neutral investors and $\delta_\pi = \frac{1}{\beta}$, becomes:

$$\begin{aligned}-\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} &= \widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1} + \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s - \beta^2\overline{R}\Theta^F\mathbb{E}_s\widehat{\eta}_{s+1} + \beta e_s^R\\ \frac{\phi}{\gamma}\left(\widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1}\right) &= w\chi\widehat{\eta}_s + \frac{\kappa}{\overline{q}}\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \beta(1-\omega)\frac{\kappa}{\overline{q}}\frac{\alpha}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\\ e_s^R &= \rho_R e_{s-1}^R + \sigma_R\varepsilon_s^R\end{aligned}$$

Combining the first two equations gives:

$$\begin{aligned}& w\chi\widehat{\eta}_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta(1-\omega)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right)\\ = \;& \frac{\phi}{\gamma}\left(-\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} - \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s + \beta^2\overline{R}\Theta^F\mathbb{E}_s\widehat{\eta}_{s+1} - \beta e_s^R\right),\end{aligned}$$

which we can re-write as

$$\left(\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\overline{R}\chi + \frac{\phi}{\gamma}\beta^2\overline{R}\Theta^F\right)\mathbb{E}_s\widehat{\eta}_{s+1} = \left(w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha} + \frac{\phi}{\gamma}\mu\chi\beta + \frac{\phi}{\gamma}\frac{\beta\delta_\theta}{1-\alpha}\right)\widehat{\eta}_s + \frac{\phi}{\gamma}\beta e_s^R.$$

Which delivers which gives

$$\mathbb{E}_s\widehat{\eta}_{s+1} = \Psi\widehat{\eta}_s - \Omega^R e_s^R,$$

where $\Psi$ is as given in the main text and

$$\Omega^R = \frac{-\phi}{\gamma\frac{\kappa}{q}\frac{\alpha(1-\omega)}{1-\alpha} + \phi\mu\beta\overline{R}\chi + \phi\beta\overline{R}\Theta^F}.$$

We again concentrate on the determinate case ($\Psi > 1$) and apply the method of undetermined

coefficients and guess a solution of the form $\widehat{\eta}_s = \Gamma_\eta^R e_s^R$. Plugging this guess into the above system of equations yields the following solution:

$$\Gamma_\eta^R = \frac{\Omega^R}{\Psi - \rho_R}. \tag{37}$$

It can now be shown that, in the determinacy region of the parameter space, the job finding rate responds negatively to contractaionary monetary policy shocks, i.e. $\Gamma_\eta^R < 0$. To see why, note the numerator of Equation (37) is negative and the denominator is positive under determinacy, since it then holds that $\Psi > 1 > \rho_R$.

Writing out the solution for $\Gamma_\eta^R$ explicitly gives:

$$\Gamma_\eta^R = \frac{-\phi}{\phi\beta\left(\frac{\delta_\theta}{1-\alpha} - \rho_R\overline{R}\Theta^F\right) + \frac{\gamma}{\beta}\frac{\kappa}{q}\frac{\alpha(1-\beta\rho_R(1-\omega))}{1-\alpha} + \left(\frac{\gamma}{\beta}w + \phi\mu\left(1-\rho_R\beta\overline{R}\right)\right)\chi}.$$

Let us now solve for the inflation rate, guessing a solution of the form $\widehat{\Pi}_s = \Gamma_\Pi^R e_s^R$. Plugging this guess into the log-linearized Euler equation gives:

$$\Gamma_\Pi^R = \frac{\beta\left(\beta\overline{R}\Theta^F\rho_R - \frac{\delta_\theta}{1-\alpha} - \chi\left(1-\rho_R\beta\overline{R}\right)\right)\Gamma_\eta^R - \beta}{1-\beta\rho_R}$$

**Belief shocks**

From Equation (35) it follows that if the equilibrium is locally determinate ($\Psi > 1$), then the only stable solution is given by $\widehat{\eta}_s = 0$ at all times. When equilibria are locally indeterminate, the solution is given by

$$\widehat{\eta}_{s+1} = \Psi\widehat{\eta}_s + \Upsilon^B\varepsilon_{s+1}^B,$$

where $\varepsilon_s^B$ is an i.i.d. belief shock with mean zero and a standard deviation normalized to one, and $\Upsilon^B$ is a parameter. Thus, in a model with only belief shocks the job finding rate follows an AR(1) process. While the magnitude of the belief shocks, captured by $\Upsilon^B$, is not pinned down in the model, the persistence of the effects of belief shocks on the job finding rate is captured by $\Psi$, and thus endogenously determined. Persistence is maximal at $\Psi = 1$, i.e. exactly at the border between the determinacy and indeterminacy region of the parameter space.

## A3. Risk-averse investors

When we log-linearized the model, we have assumed for simplicity that the asset-rich firm owners are risk neutral. The reason is that, technically, the unemployment rate becomes a state variable for inflation and the job finding rate, once we assume risk averse investors. With an additional state variable, the analytical solution of the model becomes more cumbersome, detracting from the key intuitions of the model.

Below, we use numerical simulations to compare versions with risk-neutral and risk-averse investors, showing only very small differences. We parametrize the model as follows (see Table below for the parameter values). We choose the subjective discount factor $\beta$ target a steady-state interest rate of 3 percent per annum. The coefficient of risk aversion, $\mu$, is set to 2, whereas the elasticity of substitution between goods, $\gamma$, is set to 6. To calibrate the price-stickiness parameter $\phi$, we exploit the observational equivalence between the Calvo and Rotemberg versions of the log-linearized New Keynesian model, and target an average price duration of 5 months. The home production parameter, $\vartheta$, is set to imply a 20 percent consumption drop upon unemployment.

| Parameter values (monthly model) | | |
|---|---|---|
| $\delta_\pi$ | 1.5 | Taylor rule coefficient inflation |
| $\delta_\theta$ | 0 | Taylor rule coefficient tightness |
| $\mu$ | 2 | coefficient of risk aversion |
| $\gamma$ | 6 | elasticity of substitution goods varieties |
| $\chi$ | 0 | real wage flexibility parameter |
| $\omega$ | 0.02 | separation rate |
| $\rho_A$ | 0.95 | persistence technology shock |
| $\alpha$ | 0.5 | matching function elasticity |
| $\kappa$ | 0.8667 | vacancy cost |
| $\phi$ | 145 | price adjustment cost parameter |
| $\vartheta$ | 0.8 | relative consumption of the unemployed |

The vacancy cost is parametrized to target a stead-state hiring cost of about 5 percent of the quarterly wage. We further target a monthly job finding rate of 0.3 and set the job loss rate, $\omega$, to 2 percent. The matching function elasticity parameter, $\alpha$, is set to 0.5. Regarding the monetary policy rule, we set $\delta_\pi = 1.5$ and $\delta_\theta = 0$. The persistence parameter of the technology shock is set to $\rho_A = 0.95$. For simplicity we assume sticky wages ($\chi = 0$).

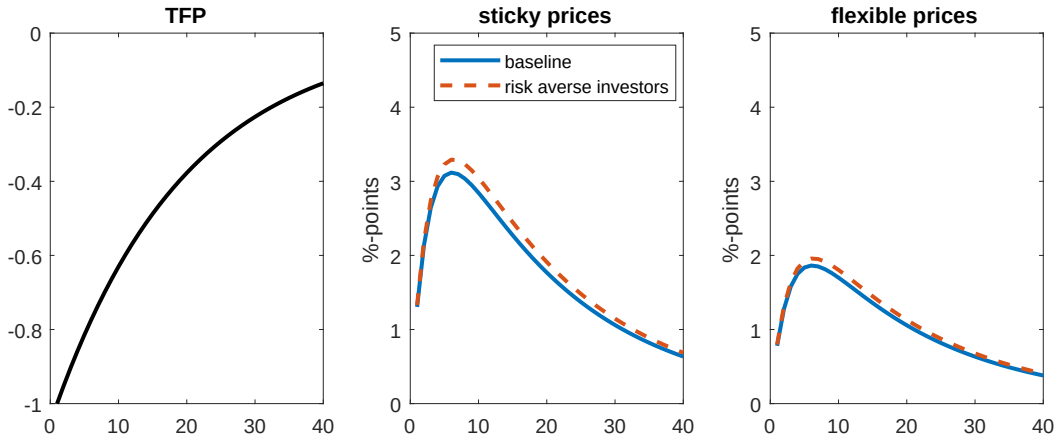



Figure 6: Responses to a positive technology shock.

The left panel of the figure below plots the response of the unemployment rate to a negative technology shock under sticky prices, in the baseline model with risk neutral investors and in the version with risk averse investors. Quantitatively, the differences are small. Next, we consider a version of the model with flexible prices (right panel). Effectively, this removes the amplification mechanism from the model, so the increase in unemployment is considerably smaller. Again, however, the differences between the baseline and the version with risk neutral investors are minor.

## A4. The Euler equation at the ZLB

Consider the setup described in Section 5.3. For simplicity, we further assume that when the economy is in the depressed (ZLB) state, the households do not expect any further shock other than that the economy returns to the normal state with a probability $p$.

In the depressed state it holds, for $x = \{\eta, \Pi\}$, that $\mathbb{E}_s x_{s+1} = p\mathbb{E}_s x_{s+1}^{ZLB} + (1-p)x$, where $x$ is the level at the indended steady state and a superscript $ZLB$ indicates that the economy remains in the depressed state. Log-linearization of this equation around the intended steady state gives $\mathbb{E}_s \widehat{x}_{s+1} = p\mathbb{E}_s \widehat{x}_{s+1}^{ZLB}$. Note further that at the ZLB, $R_s = 1$ and hence $\widehat{R}_s = -\ln \overline{R}$.

Applyying these results to the Euler equation, log-linearized around the intended steady state and as derived above, gives:

$$\left(\mu\chi\left(1-\beta\overline{R}p\right) - \beta\overline{R}\Theta^F p\right)\widehat{\eta}_s = \ln\overline{R} + p\widehat{\Pi}_s.$$

Here we have used that if the ZLB binds in period $s$ then $\mathbb{E}_s \widehat{x}_{s+1} = p\mathbb{E}_s \widehat{x}_{s+1}^{ZLB} = p\widehat{x}_s$, exploiting the fact that variables remain constant as long as the depressed state persists. The Euler equation thus defines a linear relation between $\widehat{\Pi}_s$ and $\widehat{\eta}_s$, with a slope given by:

$$\frac{d\widehat{\Pi}_s}{d\widehat{\eta}_s} = \frac{\mu\chi}{p}\left(1 - \beta\overline{R}p\right) - \beta\overline{R}\Theta^F.$$

Applying the same logic, the log-linearized Phillips Curve at the ZLB can be written as:

$$\frac{\phi}{\gamma}\left(1 - \beta p\right)\widehat{\Pi}_s = \left(w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1 - \beta\left(1-\omega\right)p\right)\right)\widehat{\eta}_s - wA_s,$$

which again defines a linear relation between $\widehat{\Pi}_s$ and $\widehat{\eta}_s$, conditional on the level of productivity $A_s$. The slope of the Phillips Curve is given by:

$$\frac{d\widehat{\Pi}_s}{d\widehat{\eta}_s} = \frac{w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1 - \beta\left(1-\omega\right)p\right)}{\frac{\phi}{\gamma}\left(1 - \beta p\right)},$$

which is always positive. Note that an (unexpected) increase in productivity ($A_s$) shifts down the Phillips Curve, i.e. it reduces inflation, conditional on a certain level of the job finding rate.

## A5. Monetary Policy and the Unemployment Trap

Here we consider the impact of systematic monetary policy on the steady-state properties of the model. Notice first that the responses of monetary policy to inflation and labor market tightness have no direct influence on the model once the nominal interest rate is constrained by the zero lower bound. Thus, we concentrate on the case in which the net nominal interest rate is positive.

For positive net nominal interest rates, the relationship between inflation and the job finding rate along the **EE** curve can be expressed as

$$\Pi = \left[\beta\overline{R}\left(\frac{\eta}{\overline{\eta}}\right)^{\delta_\theta/(1-\alpha)}\overline{\Pi}^{-\delta_\pi}\boldsymbol{\Theta}^{SS}\left(\eta\right)\right]^{-1/(\delta_\pi - 1)}$$

where $\overline{\eta} = \overline{\theta}^{1-\alpha}$. Notice that $-1/\left(\delta_\pi - 1\right) < 0$ since we imposed $\delta_\pi > 1$.

Assume first that $\delta_\theta = 0$ and consider the impact of variations in $\delta_\pi$. Assume for simplicity that $\overline{\Pi} = 1$. In this case, an increase in $\delta_\pi$ flattens the **EE** relationship tilting it around the intended steady state of the economy. Recall that existence of the unemployment trap requires

the endogenous earning risk wedge to be sufficiently countercyclical, $\partial \mathbf{\Theta}^{SS}(\eta)/\partial \eta < 0$ so that the **EE** curve is steeper than the **PC**. Minor variations in $\delta_\pi$ will have no impact on the existence of the unemployment trap but a sufficiently large value of $\delta_\pi$ will mean that this bad long run equilibrium will cease to exist. Intuitively, aggressive policy manipulates agents' expectations so that they realize that any decline in inflation will be accompanied by a sufficiently large decline in real interest rates that savings will actually decline thereby preventing the spiral towards the unemployment trap. Figure 7 below, left panel, shows one such situation.

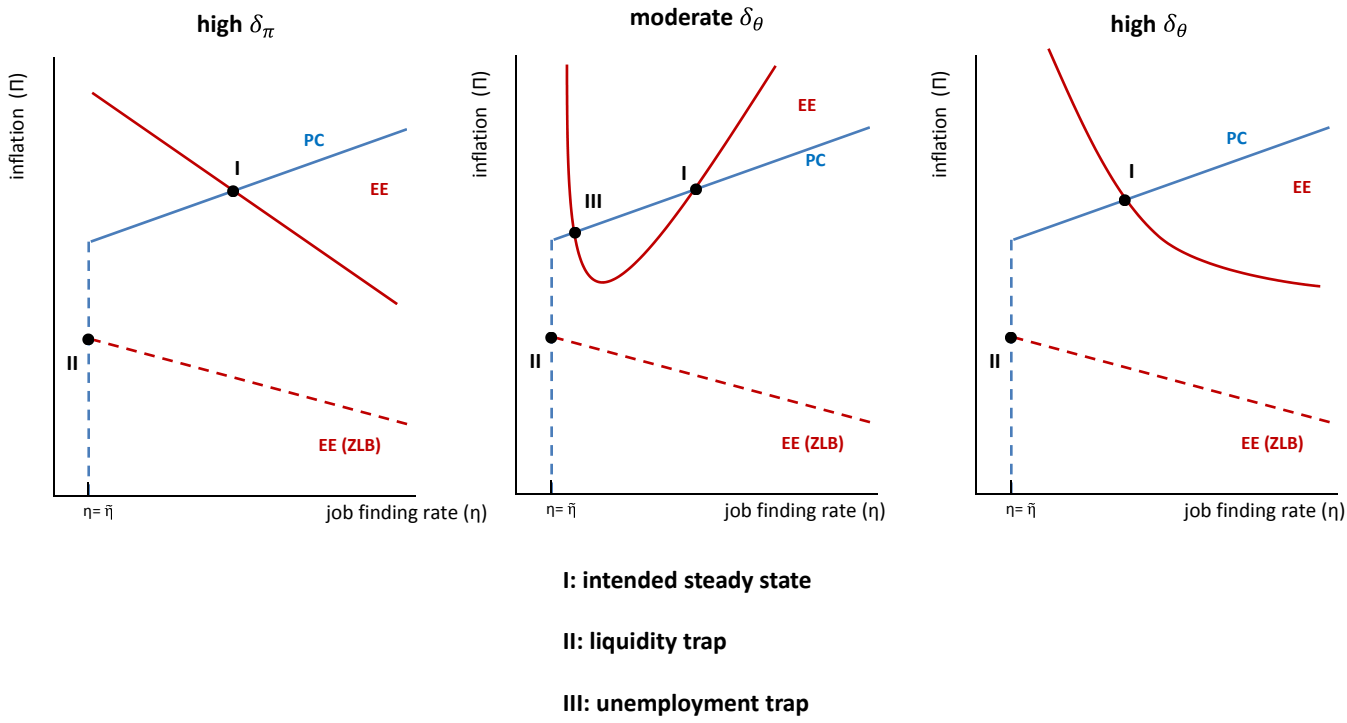
Consider now the impact of $\delta_\theta$. Notice that once $\delta_\theta > 0$, the **EE** will depend on labor market tightness and therefore on the job finding rate on top of the incomplete markets wedge, $\mathbf{\Theta}^{SS}(\eta)$. Notice also that the relationship between inflation and job finding rates along the **EE** curve implies that the **EE** curve becomes vertical as $\eta \to 0$. This implies that, if the unemployment trap exists, it is close to, but not exactly at $\eta = 0$. Marginal increases in $\delta_\theta$ will then fail to rule out the existence of the unemployment trap but make this equilibrium slightly less bad. A sufficiently large value of $\delta_\theta$ will, however, similarly to the impact of $\delta_\pi$, rule out the existence of the unemployment trap. Intutitively, when $\delta_\theta$ is large enough, the central bank can rule out the unemployment trap by signalling that any deterioration in the labor market will be accompanied by large cuts in the nominal interest rate which stimulate the economy ruling out a spiral towards the bad equilibrium. The middle and right panels of Figure 7 show these policy configurations graphically graphically.

## A6. Less extreme unemployment traps

The unemployment trap discussed above is an extreme outcome in which firms do not hire at all. However, less extreme unemployment traps are also possible, under minor modifications of the model setup. One example is the case in which monetary policy responds moderately to labor market tightness, as shown in the middle panel of Figure 7. In that case, the unemployment trap occurs at a low but positive job finding rate, i.e. unemployment is below 100 percent.

An alternative setting with a similar outcome is one in which there is some frictionless hiring. Suppose each firms receives a limited number of costless vacancies, capturing the reality that some hiring takes place via informal channels which do not require explicit recruitment costs. In the intended steady state, firms then exhaust all their costless vacancies and top them up with costly vacancies. In the unemployment trap, however, firms only use their costless vacancies. Figure 8 illustrates this case: there is still hiring in the unemployment trap, as the job finding rate drops to $\widetilde{\eta} > 0$. The associated unemployment rate is high, relative to the intended steady

Figure 7: Illustration of steady-state equilibria: alternative monetary policy rules.




state, but does not reach 100 percent.[35]

## A7. The Model with Capital Accumulation

We now consider a version of the model in which firms invest not only in vacancies, but also in physical capital. We assume a Cobb-Douglas production function: $y_{j,s} = \exp(A_s) k_{j,s}^{\gamma_k} n_{j,s}^{1-\gamma_k}$, where $\gamma_k \in [0,1]$ is the production elasticity with respect to capital. The marginal cost of production now becomes:

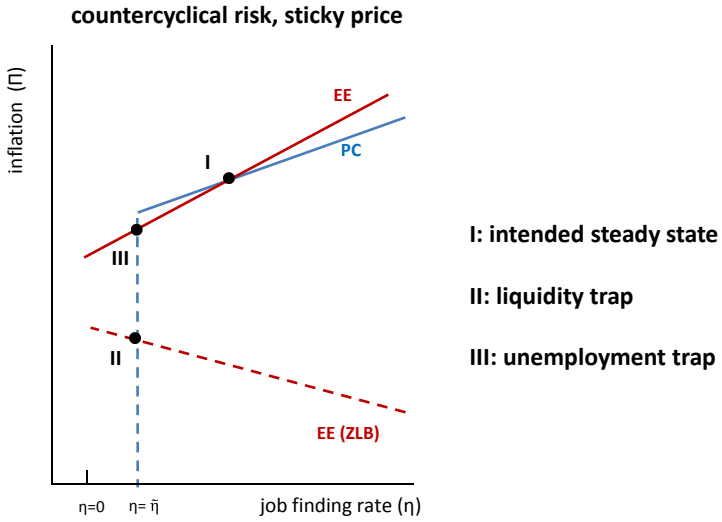
$$mc_{j,s} = \frac{1}{mpl_{j,s}} \left( w_s + \frac{\kappa}{q_s} - \lambda_{v,j,s} - (1-\omega)\, \mathbb{E}_s \Lambda_{j,s,s+1} \left\{ \frac{\kappa}{q_{s+1}} - \lambda_{v,j,s+1} \right\} \right),$$

where $mpl_{j,s}$ is the marginal product of labor, which is given by $mpl_{j,s} = (1-\gamma_k)\, y_{j,s}/n_{j,s}$. The stock of capital evolves as:

$$k_{j,s+1} = (1-\delta_k) k_{j,s} + i_{j,s},$$

[35] There might be additional equilibria in which firms choose not to even make use of the costless vacancies.

Figure 8: Illustration of steady-state equilibria: model with some costless vacancies.




where $\delta_k \in (0, 1]$ is the depreciation rate of capital and $i_{j,s}$ denotes capital investment. The Euler equation for capital investment is given by:

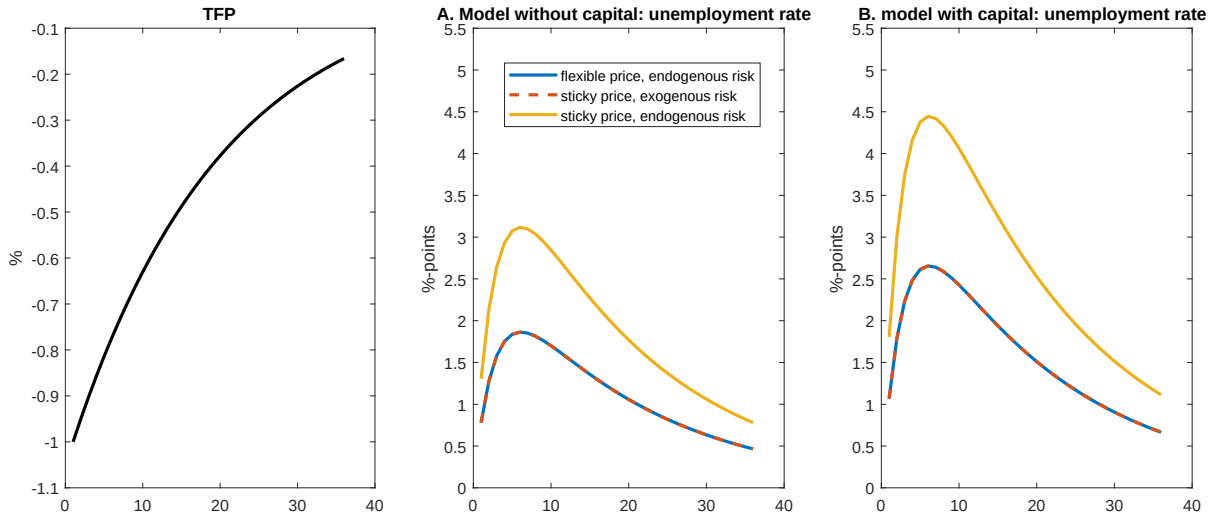
$$1 = \mathbb{E}_s \Lambda_{j,s,s+1}(1 - \delta_k + mpk_{j,s+1}),$$

where $mpk_{j,s+1} = \gamma_k y_{j,s}/k_{j,s}$ is the marginal product of capital.

We compare amplification in the model with and without capital. To this end we consider two additional versions, for both the model with and without capital. First, we consider a version with flexible prices. Second, we consider a version with sticky prices, but with exogenous unemployment risk. The latter version is obtained by assuming that those who become unemployed can only become employed again with a one-month lag. In that case, unemployment risk is purely determined by the separation rate, which is an exogenous parameter. For comparability with the baseline model, we then re-calibrate the separation rate such that the steady-state unemployment inflow probability is the same as in the baseline.

We solve the models numerically, as the model with capital can no longer be solved analytically. The calibration follows Appendix A3. In the model we capital, we further set $\gamma_k = 0.3$ and $\delta_k = 0.01$. Figure 9 displays response of the unemployment rate to a negative shock to Total Factor Productivity. The left panel shows that in the model without capital, there is substantial amplification in the baseline, relative to the versions with sticky prices and exogenous risk. The right panel shows that this amplification is still present once we introduce capital. In fact, the unemployment responses are substantially stronger than in the version without capital.

Figure 9: Responses of the unemployment rate to a TFP shock.




## A8. Nominal stickiness in unemployment benefits

In this section, we study a version of the model with nominal stickiness in the unemployment benefit. This may have stabilizing effects. Intuitively, if prices decline during a recession but benefits are nominally sticky, the real value of the benefit increases. This effects makes income risk less countercyclical, provided that prices fall during recessions. To study this case, we modify the model to include the following law of motion for the real value unemployment benefits:

$$\vartheta_t = \iota\overline{\vartheta} + (1-\iota)\frac{\vartheta_{t-1}}{1+\pi_t},$$

where $\iota \in [0,1]$ is a parameter which controls the degree of nominal stickiness and $\overline{\vartheta}$ is the steady-state level of the benefit. If we set $\iota = 1$, we obtain the baseline model with a constant real benefit. If $\iota > 0$, there is nominal stickiness in the benefit, and a decline in inflation increases the benefit. If $\iota = 0$, benefits are fully nominally rigid.

Note that in this version of the model, we obtain an extra state variable, so we solve the model numerically. We calibrate the model as in Appendix A3 (assuming a fully rigid real wage), and set $\iota = 0.2$, implying substantial nominal stickiness. Figure 10 plots the responses to a negative productivity shock in the baseline model, a version with a sticky benefit as described above (as well as sticky prices), and a version with flexible prices (but a sticky benefit). As expected, the version with a nominally sticky benefit features less amplification than the baseline (both relative to the flexible price case). The reduced amplification follows from the fact that prices to

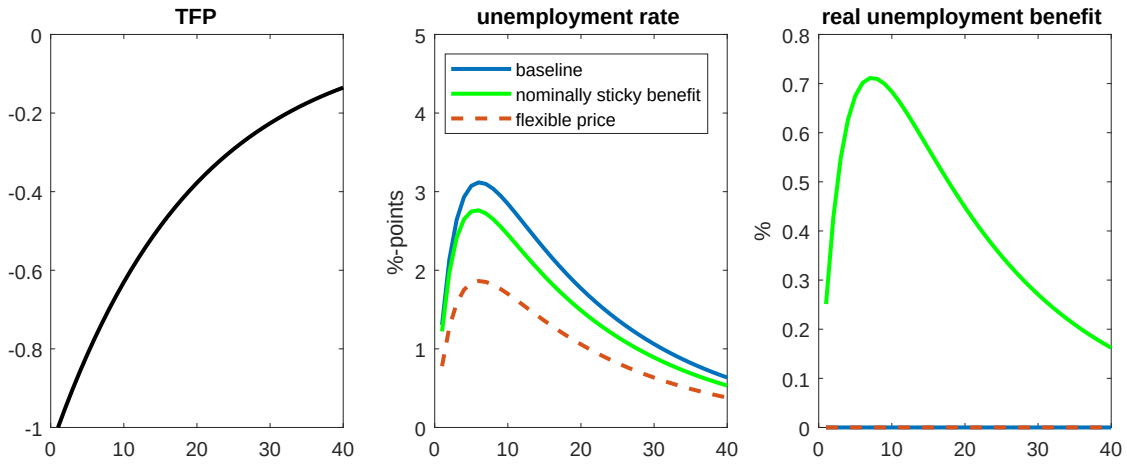



Figure 10: Responses to a positive technology shock.

decline following the productivity shock, due to the ensuing decline in aggregate demand. With a nominally sticky benefit, the decline in prices increase the real value of benefits, which dampens the increase in income risk. Therefore, unemployment increases by less than in the baseline. Nonetheless, substantial amplification is left, despite the considerable nominal stickiness in the benefit.

## A9. Pricing Risky Assets

This section explores asset pricing implications of the model. We show that the model generates a positive risk premium, but only if markets are incomplete. Intuitively, agents dislike asset with returns that co-move negatively with the probability of becoming unemployed, and hence require a discount relative to asset with acyclical returns.

For simplicity, consider the model with sticky wages ($\chi = 0$) and no sunspots. We focus on equilibria around the intended steady state. The stochastic discount factor of an employed household is given by $\Lambda_{e,s,s+1} = \beta\omega\left(1-\eta_{s+1}\right)\left(\vartheta/w\right)^{-\mu} + \beta\left(1-\omega\left(1-\eta_{s+1}\right)\right)$. Note that the period-$s$ conditional correlation between $\Lambda_{s,s+1}$ and $\eta_{s+1}$ (and hence between $\Lambda_{s,s+1}$ and $A_{s+1}$) is perfectly negative, due to the fact that $\vartheta < w$. The appendix shows that the conditional variance of the stochastic discount factor is given by:

$$Var_s\left\{\Lambda_{e,s,s+1}\right\} = \beta^2\left(\Theta^F\right)^2\Gamma_\eta^2\sigma_A^2.$$

Note that under complete markets ($\Theta^F = 0$), we obtain $Var_s\left\{\Lambda_{e,s,s+1}\right\} = 0$, i.e. the stochastic

discount factor is constant. Intuitively, when agents' income is fully insured against unemployment risk and wages are sticky, their income, and hence their desire to save, is completely constant. When markets are incomplete, the precautionary savings motive emerges and fluctuates with the cycle since the amount of unemployment and wage risk varies over the business cycle.

**Exogenous payoffs:** We now use the model to price risky assets with simple payoff structures. First, consider a risky asset that pays off $1 + A_{s+1} - \rho A_s$ in period $s+1$. We choose this payoff structure as it has the simplifying property that the expected payoff is one, while at the same time payoffs increase after an expansionary shock to productivity.

To obtain analytical tractability, we again assume that the asset is in zero net supply and that households cannot go short in the asset. As a result, the employed asset-poor households are the ones pricing the asset at the margin, whereas the other two types of households are in equilibrium at the no-short sale constraint. Krusell, Mukoyama and Smith (2011) exploit a similar setup to price risky asset under incomplete markets, but in an economy with exogenous endowments. Here, we analyze the importance of the endogenous feedback mechanism created by HANK and SAM, and study the effects of monetary policy on asset prices.

Below we show that the employed households' stochastic discount factor and the solution of the log-linearized model imply that the price of the risky asset, denoted $z_s$, is given by:

$$z_s = \mathbb{E}_s \Lambda_{e,s,s+1} - \beta \Theta^F \Gamma_\eta \sigma_A^2.$$

In the above equation, the term $\beta \Theta^F \Gamma_\eta \sigma_A^2$ is the discount relative to a riskless asset. To see this, consider a riskless asset that pays out one unit of goods in the next period regardless of the state of the world (i.e. a real bond). Again imposing the no-shortsale constraint, it follows immediately from the households' discount factor that the price of the riskless asset is given by $\mathbb{E}_s \Lambda_{e,s,s+1}$.

The above equation thus makes clear that if the endogenous earnings risk is countercyclical, i.e. $\Theta^F > 0$, there is a risk premium, which emerges despite the fact that the above equation is based on the solution of the log-linearized model.[36] Further, recall that $\Gamma_\eta$ is the response of the job finding rate to a productivity shock. The magnitude of $\Gamma_\eta$ depends on the strength of the endogenous interaction between HANK and SAM, as well as on the monetary policy rule.

[36] In representative agent models risk premia typically vanish after log-linearization since in the steady state there is no risk. Recall that in our model, by contrast, there is still idiosyncratic risk in the steady state.

By responding more aggressively to economic shocks, the central bank stabilizes the economy, reducing the strength of the precautionary savings mechanism and thereby the risk premium. Finally, note that without shocks, i.e. $\sigma_A = 0$, there is no risk premium.

**Endogenous payoffs:** Consider now another risky asset with an payoff equal to $1+\widehat{\eta}_{s+1}-\rho\widehat{\eta}_s$. Note that, again, the expected payoff is one and that the payoff is increasing in next period's job finding rate. Again, we impose the no-shortsale constraint. Below we show that the price of the asset is given by:

$$z_s = \mathbb{E}_s\Lambda_{e,s,s+1} - \beta\Theta^F\Gamma_\eta^2\sigma_A^2.$$

Note that in the return of the risky asset we now observe $\Gamma_\eta^2$ rather than $\Gamma_\eta$. This reflects the fact that the payoff of the asset is now endogenous. As a result, market frictions and monetary policy affect the risk premium via two channels: through the households' stochastic discount factor (via their unemployment risk) and through the asset payoff (via the equilibrium effects of household demand).

**Derivations:** Consider the stochastic discount factor of the employed, asset-poor households:

$$\Lambda_{e,s,s+1} = \beta\omega\left(1-\eta_{s+1}\right)\left(\vartheta/w\right)^{-\mu} + \beta\left(1-\omega\left(1-\eta_{s+1}\right)\right).$$

Given the solution, the job finding rate is –up to a first-order approximation– given by $\eta_s = \eta+\eta\Gamma_\eta A_s$. We exploit this to write the period$-s$ conditional expectation and variance of $\Lambda_{e,s,s+1}$, respectively, as:

$$\begin{aligned}
\mathbb{E}_s\Lambda_{e,s,s+1} &= \beta\omega\left(1-\mathbb{E}_s\eta_{s+1}\right)\left(\vartheta/w\right)^{-\mu} + \beta\left(1-\omega\left(1-\mathbb{E}_s\eta_{s+1}\right)\right), \\
&= \beta\omega\left(1-\eta-\rho_A\eta\Gamma_\eta A_s\right)\left(\vartheta/w\right)^{-\mu} + \beta\left(1-\omega\left(1-\eta-\rho_A\eta\Gamma_\eta A_s\right)\right),
\end{aligned}$$

and

$$\begin{aligned}
Var_s\left\{\Lambda_{e,s,s+1}\right\} &= \beta^2\omega^2\left(1-\left(\vartheta/w\right)^{-\mu}\right)^2 Var_s\left\{\eta_{s+1}\right\}, \\
&= \beta^2\omega^2\left(1-\left(\vartheta/w\right)^{-\mu}\right)^2\eta^2\Gamma_\eta^2 Var_s\left\{\rho_A A_s + \sigma_A\varepsilon_{s+1}^A\right\}, \\
&= \beta^2\omega^2\left(1-\left(\vartheta/w\right)^{-\mu}\right)^2\eta^2\Gamma_\eta^2\sigma_A^2, \\
&= \beta^2\left(\Theta^F\right)^2\Gamma_\eta^2\sigma_A^2.
\end{aligned}$$

**Exogenous payoffs:** The pricing equation for the asset that pays off $1+A_{s+1}-\rho A_s$ in period

$s+1$ reads:

$$\begin{aligned} z_s &= \mathbb{E}_s\left\{\Lambda_{e,s,s+1}\left(1+A_{s+1}-\rho_A A_s\right)\right\} \\ &= \mathbb{E}_s\Lambda_{e,s,s+1}\mathbb{E}_s\left(1+A_{s+1}-\rho_A A_s\right)+Cov_t(\Lambda_{e,s,s+1},1+A_{s+1}-\rho_A A_s) \\ &= \mathbb{E}_s\Lambda_{e,s,s+1}-\sqrt{Var_s\left\{\Lambda_{e,s,s+1}\right\}Var_s\left\{1+A_{s+1}-\rho_A A_s\right\}} \\ &= \mathbb{E}_s\Lambda_{e,s,s+1}-\beta\Theta^F\Gamma_\eta\sigma_A^2 \end{aligned}$$

where we exploited the fact that the $Cor_s\left\{\Lambda_{s,s+1},A_{s+1}\right\}=-1$, that $1+\mathbb{E}_s A_{s+1}-\rho_A A_s=1$, and that $Var_s\left\{1+A_{s+1}-\rho_A A_s\right\}=\sigma_A^2$.

**Endogenous payoffs:** Consider now another risky asset with an payoff equal to $1+\widehat{\eta}_{s+1}-\rho\widehat{\eta}_s$. The pricing equation for this asset reads:

$$\begin{aligned} z_s &= \mathbb{E}_s\left\{\Lambda_{e,s,s+1}\left(1+\widehat{\eta}_{s+1}-\rho_A\widehat{\eta}_s\right)\right\}, \\ &= \mathbb{E}_s\left\{\Lambda_{e,s,s+1}\left(1+\Gamma_\eta A_{s+1}-\rho_A\Gamma_\eta A_s\right)\right\}, \\ &= \mathbb{E}_s\Lambda_{e,s,s+1}-\sqrt{Var_s\left\{\Lambda_{e,s,s+1}\right\}Var_s\left\{1+\Gamma_\eta A_{s+1}-\rho_A\Gamma_\eta A_s\right\}}, \\ &= \mathbb{E}_s\Lambda_{e,s,s+1}-\beta\Theta^F\Gamma_\eta^2\sigma_A^2. \end{aligned}$$